

1: Suppose that 13 people are each dealt 4 cards from a standard 52-card deck. Show that it is possible for each of them to select one of their cards so that no two people have selected a card of the same rank.

2, Diestel 2.4: Moving alternatively, two players jointly construct a path in some fixed graph G . If v_1, \dots, v_n is the path constructed so far, the player to move next has to find a vertex v_{n+1} such that v_1, \dots, v_{n+1} is again a path. Whichever player cannot move loses. For which graphs G does the first player have a winning strategy, for which the second?

3, Diestel 2.10: Prove *Sperner's theorem*: in an n -set X there are never more than $\binom{n}{\lfloor n/2 \rfloor}$ subsets such that none of these contain another.

(Hint. Construct $\binom{n}{\lfloor n/2 \rfloor}$ chains covering the power set lattice of X .)

4, Diestel 2.18: Show that a graph G contains k independent edges if and only if $q(G - S) \leq |S| + |G| - 2k$ for all sets $S \subseteq V(G)$.

5, Diestel 2.24: Show that if G has two edge-disjoint spanning trees, it has a connected spanning subgraph all whose degrees are even.

6: Show that a tree T has a perfect matching if and only if $q(T - v) = 1$ for every $v \in V(T)$.
