Q1. Let $d \in \mathbb{N}$ and $V := \{0,1\}^d$; thus, V is the set of all 0–1 sequences of length d. The graph on V in which two such sequences form an edge if and only if they differ in exactly one position is called the *d-dimensional cube*. Determine the average degree, number of edges, diameter, girth and circumference of this graph.

(Hint for the circumference: induction on d.)

Q2. Show that graphs of girth at least 5 and order n have a minimum degree of $o(n)$. In other words, show that there is a function $f : \mathbb{N} \to \mathbb{N}$ such that $f(n)/n \to 0$ as $n \to \infty$ and $\delta(G) \leq f(n)$ for all such graphs G.

Q3. Show that every connected graph G contains a path of length at least $\min\{2\delta(G), |G| - 1\}$.

Q4. Determine $\kappa(G)$ and $\lambda(G)$ for $G = P^m, C^n, K^n, K_{m,n}$ and the d-dimensional cube (Exercise 2); $d, m, n \geq 3$.

Q5. Show that a tree without a vertex of degree 2 has more leaves than other vertices. Can you find a very short proof that does not use induction?

Q6. Show that every automorphism of a tree fixes a vertex or an edge.

Q7. A graph is *self-complementary* if it is isomorphic to its complement. Show that:

(a) The number of vertices in any self-complementary graph is congruent to 0 or 1 mod 4.

(b) Every self-complementary graph on $4k + 1$ vertices has a vertex of degree $2k$.

Q8. A tree is *homeomorphically irreducible* if it has no vertices of degree 2. Draw all non-isomorphic, homeomorphically irreducible trees on 10 vertices. Explain why all such trees are represented among your drawings.