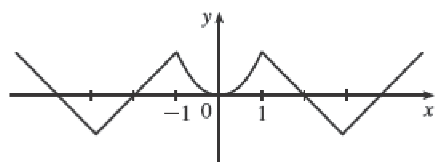
4. $f(x) = \sqrt{x^2 + x + 1}$, $[-2, 1]$. $f'(x) = \frac{1}{2}(x^2 + x + 1)^{-1/2}(2x + 1) = \dfrac{2x + 1}{2\sqrt{x^2 + x + 1}}$. $f'(x) = 0 \Rightarrow x = -\dfrac{1}{2}$.

$f'(x) > 0$ for $-\frac{1}{2} < x < 1$ and $f'(x) < 0$ for $-2 < x < -\frac{1}{2}$, so $f\left(-\frac{1}{2}\right) = \sqrt{3}/2$ is a local minimum value. Checking the endpoints, we find $f(-2) = f(1) = \sqrt{3}$. Thus, $f\left(-\frac{1}{2}\right) = \sqrt{3}/2$ is the absolute minimum value and $f(-2) = f(1) = \sqrt{3}$ is the absolute maximum value.

8. This limit has the form $\frac{0}{0}$. $\displaystyle\lim_{x \to 0} \frac{\tan 4x}{x + \sin 2x} \overset{\text{H}}{=} \lim_{x \to 0} \frac{4 \sec^2 4x}{1 + 2\cos 2x} = \frac{4(1)}{1 + 2(1)} = \frac{4}{3}$

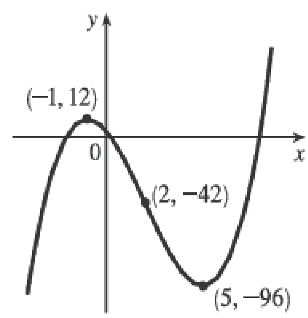
12. This limit has the form $0 \cdot \infty$. $\displaystyle\lim_{x \to \pi^-} (x - \pi) \csc x = \lim_{x \to \pi^-} \frac{x - \pi}{\sin x}$ $\left[\frac{0}{0} \text{ form}\right] \overset{\text{H}}{=} \displaystyle\lim_{x \to \pi^-} \frac{1}{\cos x} = \frac{1}{-1} = -1$

16. For $0 < x < 1$, $f'(x) = 2x$, so $f(x) = x^2 + C$. Since $f(0) = 0$, $f(x) = x^2$ on $[0, 1]$. For $1 < x < 3$, $f'(x) = -1$, so $f(x) = -x + D$. $1 = f(1) = -1 + D \Rightarrow D = 2$, so $f(x) = 2 - x$. For $x > 3$, $f'(x) = 1$, so $f(x) = x + E$. $-1 = f(3) = 3 + E \Rightarrow E = -4$, so $f(x) = x - 4$. Since f is even, its graph is symmetric about the y-axis.



20. $y = f(x) = x^3 - 6x^2 - 15x + 4$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 4$; x-intercepts: $f(x) = 0 \Rightarrow x \approx -2.09, 0.24, 7.85$ **C.** No symmetry **D.** No asymptote **E.** $f'(x) = 3x^2 - 12x - 15 = 3(x^2 - 4x - 5) = 3(x + 1)(x - 5)$, so f is increasing on $(-\infty, -1)$, decreasing on $(-1, 5)$, and increasing on $(5, \infty)$. **F.** Local maximum value $f(-1) = 12$, local minimum value $f(5) = -96$. **G.** $f''(x) = 6x - 12 = 6(x - 2)$, so f is CD on $(-\infty, 2)$ and CU on $(2, \infty)$. There is an IP at $(2, -42)$.

H.



24. $y = f(x) = \dfrac{1}{x^2} - \dfrac{1}{(x-2)^2}$ **A.** $D = \{x \mid x \neq 0, 2\}$ **B.** y-intercept: none; x-intercept: $f(x) = 0 \;\Rightarrow$

$\dfrac{1}{x^2} = \dfrac{1}{(x-2)^2} \;\Leftrightarrow\; (x-2)^2 = x^2 \;\Leftrightarrow\; x^2 - 4x + 4 = x^2 \;\Leftrightarrow\; 4x = 4 \;\Leftrightarrow\; x = 1$ **C.** No symmetry

D. $\lim\limits_{x\to 0} f(x) = \infty$ and $\lim\limits_{x\to 2} f(x) = -\infty$, so $x = 0$ and $x = 2$ are VA; $\lim\limits_{x\to\pm\infty} f(x) = 0$, so $y = 0$ is a HA

E. $f'(x) = -\dfrac{2}{x^3} + \dfrac{2}{(x-2)^3} > 0 \;\Rightarrow\; \dfrac{-(x-2)^3 + x^3}{x^3(x-2)^3} > 0 \;\Leftrightarrow\; \dfrac{-x^3 + 6x^2 - 12x + 8 + x^3}{x^3(x-2)^3} > 0 \;\Leftrightarrow$

$\dfrac{2(3x^2 - 6x + 4)}{x^3(x-2)^3} > 0$. The numerator is positive (the discriminant of the quadratic is negative), so $f'(x) > 0$ if $x < 0$ or

$x > 2$, and hence, f is increasing on $(-\infty, 0)$ and $(2, \infty)$ and decreasing on $(0, 2)$.

F. No local extreme values **G.** $f''(x) = \dfrac{6}{x^4} - \dfrac{6}{(x-2)^4} > 0 \;\Rightarrow$

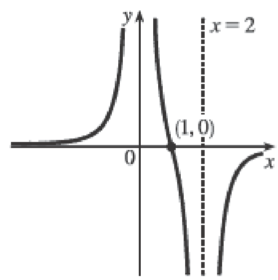
$\dfrac{(x-2)^4 - x^4}{x^4(x-2)^4} > 0 \;\Leftrightarrow\; \dfrac{x^4 - 8x^3 + 24x^2 - 32x + 16 - x^4}{x^4(x-2)^4} > 0 \;\Leftrightarrow$

$\dfrac{-8(x^3 - 3x^2 + 4x - 2)}{x^4(x-2)^4} > 0 \;\Leftrightarrow\; \dfrac{-8(x-1)(x^2 - 2x + 2)}{x^4(x-2)^4} > 0$. So f'' is

positive for $x < 1$ $[x \neq 0]$ and negative for $x > 1$ $[x \neq 2]$. Thus, f is CU on

$(-\infty, 0)$ and $(0, 1)$ and f is CD on $(1, 2)$ and $(2, \infty)$. IP at $(1, 0)$

H.



28. $y = f(x) = \sqrt[3]{x^2 + 1}$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 1$; $f(x) > 0$, so there is no x-intercept **C.** $f(-x) = f(x)$, so the curve is symmetric about the y-axis. **D.** No asymptote **E.** $f'(x) = \frac{1}{3}(x^2+1)^{-2/3}(2x) = \dfrac{2x}{3(x^2+1)^{2/3}} > 0 \;\Leftrightarrow$

$x > 0$ and $f'(x) < 0 \;\Leftrightarrow\; x < 0$, so f is increasing on $(0, \infty)$ and decreasing on $(-\infty, 0)$.

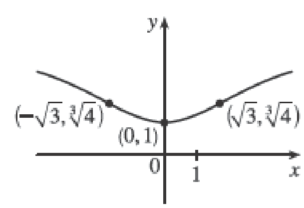
F. Local minimum value $f(0) = 1$

G. $f''(x) = \dfrac{3(x^2+1)^{2/3}(2) - 2x(3)\frac{2}{3}(x^2+1)^{-1/3}(2x)}{[3(x^2+1)^{2/3}]^2} = \dfrac{2(x^2+1)^{-1/3}\left[3(x^2+1) - 4x^2\right]}{9(x^2+1)^{4/3}} = \dfrac{2(3 - x^2)}{9(x^2+1)^{5/3}}$.

$f''(x) > 0 \;\Leftrightarrow\; -\sqrt{3} < x < \sqrt{3}$ and $f''(x) < 0 \;\Leftrightarrow\; x < -\sqrt{3}$ and $x > \sqrt{3}$, so f is CU on $\left(-\sqrt{3}, \sqrt{3}\right)$ and CD on $\left(-\infty, -\sqrt{3}\right)$ and $\left(\sqrt{3}, \infty\right)$.

There are inflection points at $\left(\pm\sqrt{3}, f\left(\pm\sqrt{3}\right)\right) = \left(\pm\sqrt{3}, \sqrt[3]{4}\right)$.

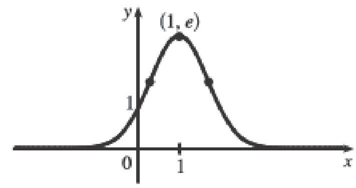
H.



32. $y = f(x) = e^{2x-x^2}$ **A.** $D = \mathbb{R}$ **B.** y-intercept 1; no x-intercept **C.** No symmetry **D.** $\lim\limits_{x\to\pm\infty} e^{2x-x^2} = 0$, so $y = 0$ is a HA. **E.** $y = f(x) = e^{2x-x^2} \Rightarrow f'(x) = 2(1-x)e^{2x-x^2} > 0 \Leftrightarrow x < 1$, so f is increasing on $(-\infty, 1)$ and decreasing on $(1, \infty)$. **F.** $f(1) = e$ is a local and absolute maximum value.

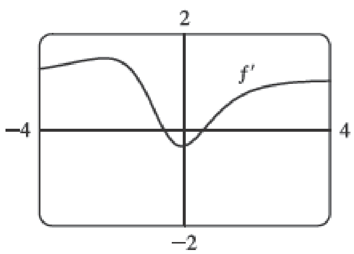
G. $f''(x) = 2\left(2x^2 - 4x + 1\right)e^{2x-x^2} = 0 \Leftrightarrow x = 1 \pm \frac{\sqrt{2}}{2}$.
$f''(x) > 0 \Leftrightarrow x < 1 - \frac{\sqrt{2}}{2}$ or $x > 1 + \frac{\sqrt{2}}{2}$, so f is CU on $\left(-\infty, 1 - \frac{\sqrt{2}}{2}\right)$ and $\left(1 + \frac{\sqrt{2}}{2}, \infty\right)$, and CD on $\left(1 - \frac{\sqrt{2}}{2}, 1 + \frac{\sqrt{2}}{2}\right)$. IP at $\left(1 \pm \frac{\sqrt{2}}{2}, \sqrt{e}\right)$

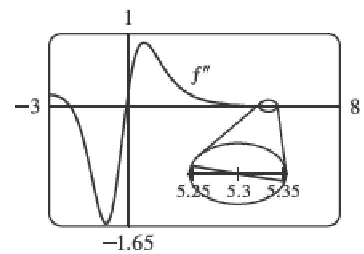
H.



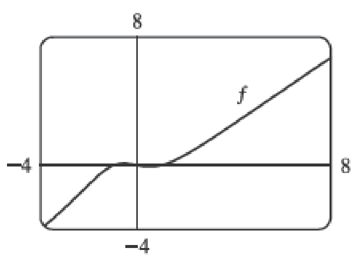
36. $f(x) = \dfrac{x^3 - x}{x^2 + x + 3} \Rightarrow f'(x) = \dfrac{x^4 + 2x^3 + 10x^2 - 3}{(x^2 + x + 3)^2} \Rightarrow f''(x) = \dfrac{-6(x^3 - 3x^2 - 12x - 1)}{(x^2 + x + 3)^3}$.

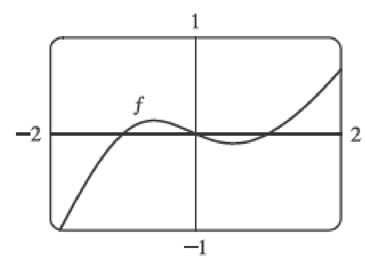
$f(x) = 0 \Leftrightarrow x = \pm 1$; $f'(x) = 0 \Leftrightarrow x \approx -0.57, 0.52$; $f''(x) = 0 \Leftrightarrow x \approx -2.21, -0.09, 5.30$.



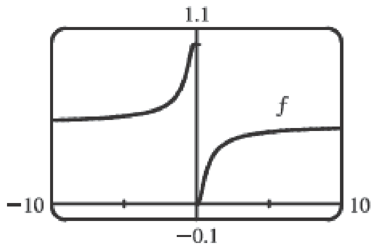


From the graphs of f' and f'', it appears that f is increasing on $(-\infty, -0.57)$ and $(0.52, \infty)$ and decreasing on $(-0.57, 0.52)$; f has a local maximum of about $f(-0.57) = 0.14$ and a local minimum of about $f(0.52) = -0.10$; f is CU on $(-\infty, -2.21)$ and $(-0.09, 5.30)$, and CD on $(-2.21, -0.09)$ and $(5.30, \infty)$; and f has inflection points at about $(-2.21, -1.52)$, $(-0.09, 0.03)$, and $(5.30, 3.95)$.





40. (a)



(b) $f(x) = \dfrac{1}{1+e^{1/x}}$.

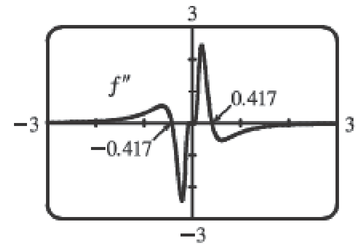
$\lim\limits_{x\to\infty} f(x) = \dfrac{1}{1+1} = \dfrac{1}{2}$, $\lim\limits_{x\to-\infty} f(x) = \dfrac{1}{1+1} = \dfrac{1}{2}$,

as $x \to 0^+$, $1/x \to \infty$, so $e^{1/x} \to \infty \;\Rightarrow\; \lim\limits_{x\to 0^+} f(x) = 0$,

as $x \to 0^-$, $1/x \to -\infty$, so $e^{1/x} \to 0 \;\Rightarrow\; \lim\limits_{x\to 0^-} f(x) = \dfrac{1}{1+0} = 1$

(c) From the graph of f, estimates for the IP are $(-0.4, 0.9)$ and $(0.4, 0.08)$.

(d) $f''(x) = -\dfrac{e^{1/x}\left[e^{1/x}(2x-1)+2x+1\right]}{x^4\left(e^{1/x}+1\right)^3}$



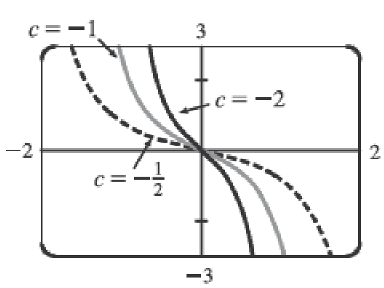
(e) From the graph, we see that f'' changes sign at $x = \pm 0.417$ ($x = 0$ is not in the domain of f). IP are approximately $(0.417, 0.083)$ and $(-0.417, 0.917)$.

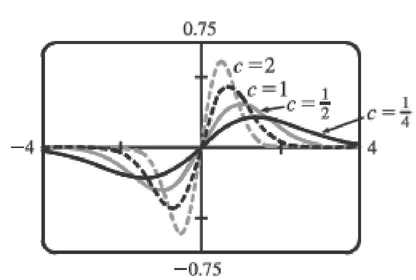
44. We exclude the case $c = 0$, since in that case $f(x) = 0$ for all x. To find the maxima and minima, we differentiate:

$$f(x) = cxe^{-cx^2} \quad\Rightarrow\quad f'(x) = c\left[xe^{-cx^2}(-2cx) + e^{-cx^2}(1)\right] = ce^{-cx^2}(-2cx^2+1)$$

This is 0 where $-2cx^2 + 1 = 0 \;\Leftrightarrow\; x = \pm 1/\sqrt{2c}$. So if $c > 0$, there are two maxima or minima, whose x-coordinates approach 0 as c increases. The negative root gives a minimum and the positive root gives a maximum, by the First Derivative Test. By substituting back into the equation, we see that $f\left(\pm 1/\sqrt{2c}\right) = c\left(\pm 1/\sqrt{2c}\right) e^{-c\left(\pm 1/\sqrt{2c}\right)^2} = \pm\sqrt{c/2e}$. So as c increases, the extreme points become more pronounced. Note that if $c > 0$, then $\lim\limits_{x\to\pm\infty} f(x) = 0$. If $c < 0$, then there are no extreme values, and $\lim\limits_{x\to\pm\infty} f(x) = \mp\infty$.

To find the points of inflection, we differentiate again: $f'(x) = ce^{-cx^2}(-2cx^2+1) \;\Rightarrow$ $f''(x) = c\left[e^{-cx^2}(-4cx) + (-2cx^2+1)(-2cxe^{-cx^2})\right] = -2c^2xe^{-cx^2}(3-2cx^2)$. This is 0 at $x = 0$ and where $3 - 2cx^2 = 0 \;\Leftrightarrow\; x = \pm\sqrt{3/(2c)} \;\Rightarrow\;$ IP at $\left(\pm\sqrt{3/(2c)}, \pm\sqrt{3c/2}\,e^{-3/2}\right)$. If $c > 0$ there are three inflection points, and as c increases, the x-coordinates of the nonzero inflection points approach 0. If $c < 0$, there is only one inflection point, the origin.





48. Since the point $(1,3)$ is on the curve $y = ax^3 + bx^2$, we have $3 = a(1)^3 + b(1)^2 \Rightarrow 3 = a + b$ **(1)**.

$y' = 3ax^2 + 2bx \Rightarrow y'' = 6ax + 2b$. $y'' = 0$ [for inflection points] $\Leftrightarrow x = \dfrac{-2b}{6a} = -\dfrac{b}{3a}$. Since we want $x = 1$,

$1 = -\dfrac{b}{3a} \Rightarrow b = -3a$. Combining with **(1)** gives us $3 = a - 3a \Leftrightarrow 3 = -2a \Leftrightarrow a = -\frac{3}{2}$. Hence,

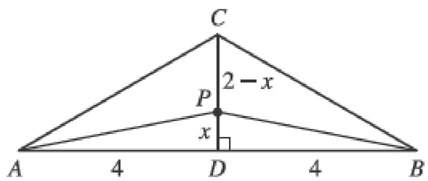
$b = -3\left(-\frac{3}{2}\right) = \frac{9}{2}$ and the curve is $y = -\frac{3}{2}x^3 + \frac{9}{2}x^2$.

52. On the hyperbola $xy = 8$, if $d(x)$ is the distance from the point $(x, y) = (x, 8/x)$ to the point $(3, 0)$, then

$[d(x)]^2 = (x-3)^2 + 64/x^2 = f(x)$. $f'(x) = 2(x-3) - 128/x^3 = 0 \Rightarrow x^4 - 3x^3 - 64 = 0 \Rightarrow$

$(x-4)\left(x^3 + x^2 + 4x + 16\right) = 0 \Rightarrow x = 4$ since the solution must have $x > 0$. Then $y = \frac{8}{4} = 2$, so the point is $(4, 2)$.

56.



If $|CD| = 2$, the last part of $L(x)$ changes from $(5 - x)$ to $(2 - x)$ with $0 \le x \le 2$. But we still get $L'(x) = 0 \Leftrightarrow x = \frac{4}{\sqrt{3}}$, which isn't in the interval $[0, 2]$. Now $L(0) = 10$ and $L(2) = 2\sqrt{20} = 4\sqrt{5} \approx 8.9$. The minimum occurs when $P = C$.