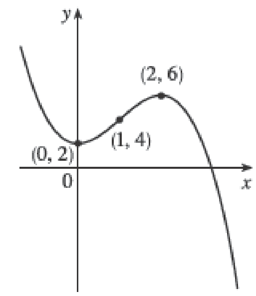
2. $y = f(x) = 2 + 3x^2 - x^3$ **A.** $D = \mathbb{R}$ **B.** y-intercept $= f(0) = 2$ **C.** No symmetry **D.** No asymptote **E.** $f'(x) = 6x - 3x^2 = 3x(2 - x) > 0 \Leftrightarrow 0 < x < 2$, so f is increasing on $(0, 2)$ and decreasing on $(-\infty, 0)$ and $(2, \infty)$.

F. Local maximum value $f(2) = 6$, local minimum value $f(0) = 2$

G. $f''(x) = 6 - 6x = 6(1 - x) > 0 \Leftrightarrow x < 1$, so f is CU on $(-\infty, 1)$ and CD on $(1, \infty)$. IP at $(1, 4)$

H.



8. $y = f(x) = (4 - x^2)^5$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 4^5 = 1024$; x-intercepts: ± 2 **C.** $f(-x) = f(x) \Rightarrow$ f is even; the curve is symmetric about the y-axis. **D.** No asymptote **E.** $f'(x) = 5(4 - x^2)^4(-2x) = -10x(4 - x^2)^4$, so for $x \neq \pm 2$ we have $f'(x) > 0 \Leftrightarrow x < 0$ and $f'(x) < 0 \Leftrightarrow x > 0$. Thus, f is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$. **F.** Local maximum value $f(0) = 1024$

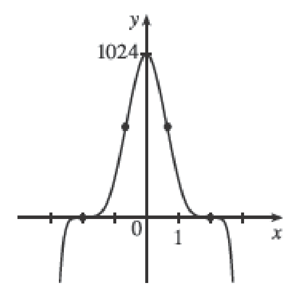
G. $f''(x) = -10x \cdot 4(4 - x^2)^3(-2x) + (4 - x^2)^4(-10)$

$$= -10(4 - x^2)^3[-8x^2 + 4 - x^2] = -10(4 - x^2)^3(4 - 9x^2)$$

so $f''(x) = 0 \Leftrightarrow x = \pm 2, \pm\frac{2}{3}$. $f''(x) > 0 \Leftrightarrow -2 < x < -\frac{2}{3}$ and $\frac{2}{3} < x < 2$ and $f''(x) < 0 \Leftrightarrow x < -2, -\frac{2}{3} < x < \frac{2}{3}$, and $x > 2$, so f is CU on $(-\infty, 2)$, $\left(-\frac{2}{3}, \frac{2}{3}\right)$, and $(2, \infty)$, and CD on $\left(-2, -\frac{2}{3}\right)$ and $\left(\frac{2}{3}, 2\right)$.

IP at $(\pm 2, 0)$ and $\left(\pm\frac{2}{3}, \left(\frac{32}{9}\right)^5\right) \approx (\pm 0.67, 568.25)$

H.



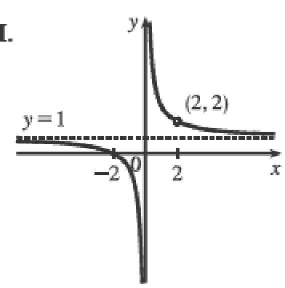
10. $y = f(x) = \dfrac{x^2 - 4}{x^2 - 2x} = \dfrac{(x + 2)(x - 2)}{x(x - 2)} = \dfrac{x + 2}{x} = 1 + \dfrac{2}{x}$ for $x \neq 2$. There is a hole in the graph at $(2, 2)$.

A. $D = \{x \mid x \neq 0, 2\} = (-\infty, 0) \cup (0, 2) \cup (2, \infty)$ **B.** y-intercept: none; x-intercept: -2 **C.** No symmetry

D. $\displaystyle\lim_{x \to \pm\infty} \frac{x + 2}{x} = 1$, so $y = 1$ is a HA. $\displaystyle\lim_{x \to 0^-} \frac{x + 2}{x} = -\infty$, $\displaystyle\lim_{x \to 0^+} \frac{x + 2}{x} = \infty$, so $x = 0$ is a VA. **E.** $f'(x) = -2/x^2 < 0$ $[x \neq 0, 2]$ so f is decreasing on $(-\infty, 0)$, $(0, 2)$, and $(2, \infty)$. **F.** No extrema

G. $f''(x) = 4/x^3 > 0 \Leftrightarrow x > 0$, so f is CU on $(0, 2)$ and $(2, \infty)$ and CD on $(-\infty, 0)$. No IP

H.



14. $y = f(x) = x^2/(x^2+9)$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 0$; x-intercept: $f(x) = 0 \;\Leftrightarrow\; x = 0$

C. $f(-x) = f(x)$, so f is even and symmetric about the y-axis. **D.** $\lim\limits_{x\to\pm\infty}\left[x^2/(x^2+9)\right] = 1$, so $y = 1$ is a HA; no VA

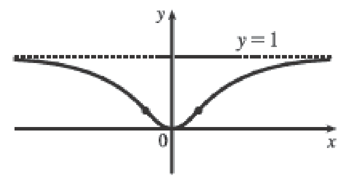
E. $f'(x) = \dfrac{(x^2+9)(2x) - x^2(2x)}{(x^2+9)^2} = \dfrac{18x}{(x^2+9)^2} > 0 \;\Leftrightarrow\; x > 0$, so f is increasing on $(0,\infty)$

and decreasing on $(-\infty, 0)$. **F.** Local minimum value $f(0) = 0$; no local maximum

G. $f''(x) = \dfrac{(x^2+9)^2(18) - 18x\cdot 2(x^2+9)\cdot 2x}{[(x^2+9)^2]^2} = \dfrac{18(x^2+9)[(x^2+9)-4x^2]}{(x^2+9)^4} = \dfrac{18(9-3x^2)}{(x^2+9)^3}$

$$= \frac{-54(x+\sqrt{3})(x-\sqrt{3})}{(x^2+9)^3} > 0 \;\Leftrightarrow\; -\sqrt{3} < x < \sqrt{3}$$

H.



so f is CU on $(-\sqrt{3}, \sqrt{3})$ and CD on $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$.

There are two inflection points: $\left(\pm\sqrt{3}, \frac{1}{4}\right)$.

20. $y = f(x) = \dfrac{x^3}{x-2} = x^2 + 2x + 4 + \dfrac{8}{x-2}$ [by long division] **A.** $D = (-\infty, 2)\cup(2,\infty)$ **B.** x-intercept $= 0$,

y-intercept $= f(0) = 0$ **C.** No symmetry **D.** $\lim\limits_{x\to 2^-}\dfrac{x^3}{x-2} = -\infty$ and $\lim\limits_{x\to 2^+}\dfrac{x^3}{x-2} = \infty$, so $x = 2$ is a VA.

There are no horizontal or slant asymptotes. *Note:* Since $\lim\limits_{x\to\pm\infty}\dfrac{8}{x-2} = 0$, the parabola $y = x^2 + 2x + 4$ is approached asymptotically as $x \to \pm\infty$.

E. $f'(x) = \dfrac{(x-2)(3x^2) - x^3(1)}{(x-2)^2} = \dfrac{x^2[3(x-2)-x]}{(x-2)^2} = \dfrac{x^2(2x-6)}{(x-2)^2} = \dfrac{2x^2(x-3)}{(x-2)^2} > 0 \;\Leftrightarrow\; x > 3$ and

$f'(x) < 0 \;\Leftrightarrow\; x < 0$ or $0 < x < 2$ or $2 < x < 3$, so f is increasing on $(3,\infty)$ and f is decreasing on $(-\infty, 2)$ and $(2, 3)$.

F. Local minimum value $f(3) = 27$, no local maximum value **G.** $f'(x) = 2\dfrac{x^3 - 3x^2}{(x-2)^2} \;\Rightarrow$

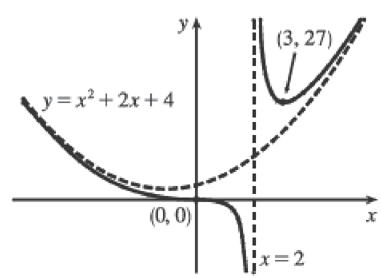
$$f''(x) = 2\frac{(x-2)^2(3x^2-6x) - (x^3-3x^2)2(x-2)}{[(x-2)^2]^2}$$

$$= 2\frac{(x-2)x[(x-2)(3x-6) - (x^2-3x)2]}{(x-2)^4}$$

$$= \frac{2x(3x^2 - 12x + 12 - 2x^2 + 6x)}{(x-2)^3}$$

$$= \frac{2x(x^2 - 6x + 12)}{(x-2)^3} > 0 \;\Leftrightarrow$$

H.



$x < 0$ or $x > 2$, so f is CU on $(-\infty, 0)$ and $(2,\infty)$, and f is CD on $(0, 2)$. IP at $(0, 0)$

28. $y = f(x) = x/\sqrt{x^2 - 1}$ **A.** $D = (-\infty, -1) \cup (1, \infty)$ **B.** No intercepts **C.** $f(-x) = -f(x)$, so f is odd; the graph is symmetric about the origin. **D.** $\lim_{x \to \infty} \frac{x}{\sqrt{x^2 - 1}} = 1$ and $\lim_{x \to -\infty} \frac{x}{\sqrt{x^2 - 1}} = -1$, so $y = \pm 1$ are HA.

$\lim_{x \to 1^+} f(x) = +\infty$ and $\lim_{x \to -1^-} f(x) = -\infty$, so $x = \pm 1$ are VA.

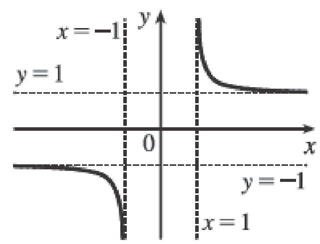
E. $f'(x) = \dfrac{\sqrt{x^2 - 1} - x \cdot \dfrac{x}{\sqrt{x^2 - 1}}}{[(x^2 - 1)^{1/2}]^2} = \dfrac{x^2 - 1 - x^2}{(x^2 - 1)^{3/2}} = \dfrac{-1}{(x^2 - 1)^{3/2}} < 0$, so f is decreasing on $(-\infty, -1)$ and $(1, \infty)$.

F. No extreme values

G. $f''(x) = (-1)\left(-\frac{3}{2}\right)(x^2 - 1)^{-5/2} \cdot 2x = \dfrac{3x}{(x^2 - 1)^{5/2}}$.

$f''(x) < 0$ on $(-\infty, -1)$ and $f''(x) > 0$ on $(1, \infty)$, so f is CD on $(-\infty, -1)$ and CU on $(1, \infty)$. No IP

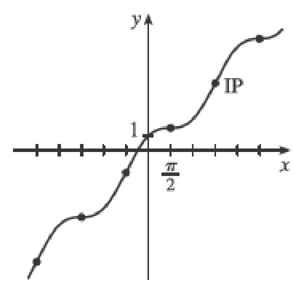
H.



34. $y = f(x) = x + \cos x$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 1$; the x-intercept is about -0.74 and can be found using Newton's method **C.** No symmetry **D.** No asymptote **E.** $f'(x) = 1 - \sin x > 0$ except for $x = \frac{\pi}{2} + 2n\pi$, so f is increasing on $\mathbb{R}$. **F.** No local extrema

G. $f''(x) = -\cos x$. $f''(x) > 0 \Rightarrow -\cos x > 0 \Rightarrow \cos x < 0 \Rightarrow$ x is in $\left(\frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi\right)$ and $f''(x) < 0 \Rightarrow$ x is in $\left(-\frac{\pi}{2} + 2n\pi, \frac{\pi}{2} + 2n\pi\right)$, so f is CU on $\left(\frac{\pi}{2} + 2n\pi, \frac{3\pi}{2} + 2n\pi\right)$ and CD on $\left(-\frac{\pi}{2} + 2n\pi, \frac{\pi}{2} + 2n\pi\right)$. IP at $\left(\frac{\pi}{2} + n\pi, f\left(\frac{\pi}{2} + n\pi\right)\right) = \left(\frac{\pi}{2} + n\pi, \frac{\pi}{2} + n\pi\right)$ [on the line $y = x$]

H.



40. $y = f(x) = \dfrac{\sin x}{2 + \cos x}$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 0$; x-intercepts: $f(x) = 0 \Leftrightarrow \sin x = 0 \Leftrightarrow x = n\pi$

C. $f(-x) = -f(x)$, so the curve is symmetric about the origin. f is periodic with period 2π, so we determine **E–G** for $0 \le x \le 2\pi$. **D.** No asymptote

E. $f'(x) = \dfrac{(2 + \cos x)\cos x - \sin x(-\sin x)}{(2 + \cos x)^2} = \dfrac{2\cos x + \cos^2 x + \sin^2 x}{(2 + \cos x)^2} = \dfrac{2\cos x + 1}{(2 + \cos x)^2}$.

$f'(x) > 0 \Leftrightarrow 2\cos x + 1 > 0 \Leftrightarrow \cos x > -\frac{1}{2} \Leftrightarrow x$ is in $\left(0, \frac{2\pi}{3}\right)$ or $\left(\frac{4\pi}{3}, 2\pi\right)$, so f is increasing on $\left(0, \frac{2\pi}{3}\right)$ and $\left(\frac{4\pi}{3}, 2\pi\right)$, and f is decreasing on $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$.

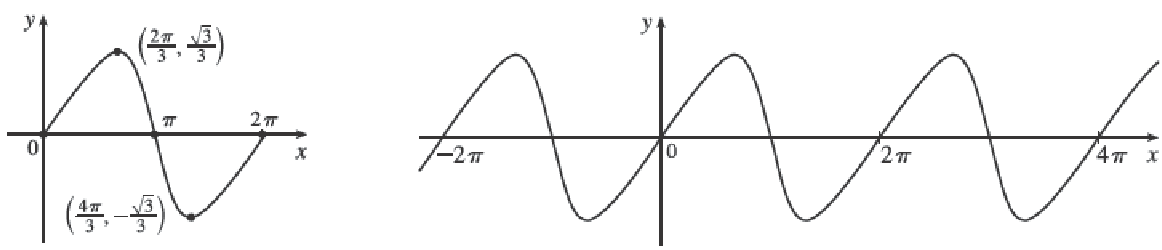
F. Local maximum value $f\left(\frac{2\pi}{3}\right) = \dfrac{\sqrt{3}/2}{2 - (1/2)} = \dfrac{\sqrt{3}}{3}$ and local minimum value $f\left(\frac{4\pi}{3}\right) = \dfrac{-\sqrt{3}/2}{2 - (1/2)} = -\dfrac{\sqrt{3}}{3}$

G. $f''(x) = \dfrac{(2 + \cos x)^2(-2\sin x) - (2\cos x + 1)2(2 + \cos x)(-\sin x)}{[(2 + \cos x)^2]^2}$

$$= \frac{-2\sin x\,(2 + \cos x)[(2 + \cos x) - (2\cos x + 1)]}{(2 + \cos x)^4} = \frac{-2\sin x\,(1 - \cos x)}{(2 + \cos x)^3}$$

$f''(x) > 0 \Leftrightarrow -2\sin x > 0 \Leftrightarrow \sin x < 0 \Leftrightarrow x$ is in $(\pi, 2\pi)$ [f is CU] and $f''(x) < 0 \Leftrightarrow x$ is in $(0, \pi)$ [f is CD]. The inflection points are $(0, 0)$, $(\pi, 0)$, and $(2\pi, 0)$.

H.



48. $y = f(x) = e^x/x^2$ **A.** $D = (-\infty, 0) \cup (0, \infty)$ **B.** No intercept **C.** No symmetry **D.** $\displaystyle\lim_{x \to -\infty} \frac{e^x}{x^2} = 0$, so $y = 0$ is HA. $\displaystyle\lim_{x \to 0} \frac{e^x}{x^2} = \infty$, so $x = 0$ is a VA. **E.** $f'(x) = \dfrac{x^2 e^x - e^x(2x)}{(x^2)^2} = \dfrac{xe^x(x - 2)}{x^4} = \dfrac{e^x(x - 2)}{x^3} > 0 \Leftrightarrow x < 0$ or $x > 2$, so f is increasing on $(-\infty, 0)$ and $(2, \infty)$, and f is decreasing on $(0, 2)$.

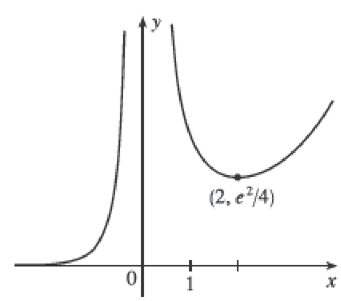
F. Local minimum value $f(2) = e^2/4 \approx 1.85$, no local maximum value

G. $f''(x) = \dfrac{x^3[e^x(1) + (x - 2)e^x] - e^x(x - 2)(3x^2)}{(x^3)^2}$

$$= \frac{x^2 e^x[x(x - 1) - 3(x - 2)]}{x^6} = \frac{e^x(x^2 - 4x + 6)}{x^4} > 0$$

for all x in the domain of f; that is, f is CU on $(-\infty, 0)$ and $(0, \infty)$. No IP

H.



52. $y = f(x) = \dfrac{\ln x}{x^2}$ **A.** $D = (0, \infty)$ **B.** y-intercept: none; x-intercept: $f(x) = 0 \iff \ln x = 0 \iff x = 1$

C. No symmetry **D.** $\lim_{x \to 0^+} f(x) = -\infty$, so $x = 0$ is a VA; $\lim_{x \to \infty} \dfrac{\ln x}{x^2} \overset{H}{=} \lim_{x \to \infty} \dfrac{1/x}{2x} = 0$, so $y = 0$ is a HA.

E. $f'(x) = \dfrac{x^2(1/x) - (\ln x)(2x)}{(x^2)^2} = \dfrac{x(1 - 2\ln x)}{x^4} = \dfrac{1 - 2\ln x}{x^3}$. $f'(x) > 0 \iff 1 - 2\ln x > 0 \iff \ln x < \frac{1}{2} \Rightarrow$

$0 < x < e^{1/2}$ and $f'(x) < 0 \Rightarrow x > e^{1/2}$, so f is increasing on $\left(0, \sqrt{e}\right)$ and decreasing on $\left(\sqrt{e}, \infty\right)$.

F. Local maximum value $f(e^{1/2}) = \dfrac{1/2}{e} = \dfrac{1}{2e}$

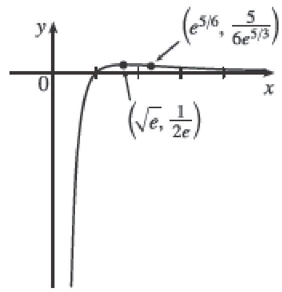
G. $f''(x) = \dfrac{x^3(-2/x) - (1 - 2\ln x)(3x^2)}{(x^3)^2}$

$= \dfrac{x^2[-2 - 3(1 - 2\ln x)]}{x^6} = \dfrac{-5 + 6\ln x}{x^4}$

$f''(x) > 0 \iff -5 + 6\ln x > 0 \iff \ln x > \frac{5}{6} \Rightarrow x > e^{5/6}$ [f is CU]

and $f''(x) < 0 \iff 0 < x < e^{5/6}$ [f is CD]. IP at $(e^{5/6}, 5/(6e^{5/3}))$

H.

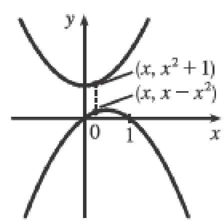


2. The two numbers are $x + 100$ and x. Minimize $f(x) = (x + 100)x = x^2 + 100x$. $f'(x) = 2x + 100 = 0 \Rightarrow x = -50$. Since $f''(x) = 2 > 0$, there is an absolute minimum at $x = -50$. The two numbers are 50 and -50.

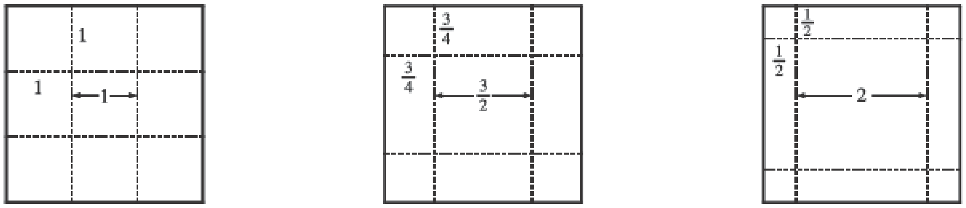
6. Let the vertical distance be given by

$v(x) = (x^2 + 1) - (x - x^2) = 2x^2 - x + 1$. $v'(x) = 4x - 1 = 0 \iff$

$x = \frac{1}{4}$. $v'(x) < 0$ for $x < \frac{1}{4}$ and $v'(x) > 0$ for $x > \frac{1}{4}$, so there is an absolute minimum at $x = \frac{1}{4}$. The minimum distance is $v\left(\frac{1}{4}\right) = \frac{1}{8} - \frac{1}{4} + 1 = \frac{7}{8}$.

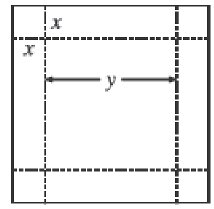


12. (a)



The volumes of the resulting boxes are 1, 1.6875, and 2 ft^3. There appears to be a maximum volume of at least 2 ft^3.

(b) Let x denote the length of the side of the square being cut out. Let y denote the length of the base.



(c) Volume $V = \text{length} \times \text{width} \times \text{height} \quad \Rightarrow \quad V = y \cdot y \cdot x = xy^2$

(d) Length of cardboard $= 3 \quad \Rightarrow \quad x + y + x = 3 \quad \Rightarrow \quad y + 2x = 3$

(e) $y + 2x = 3 \quad \Rightarrow \quad y = 3 - 2x \quad \Rightarrow \quad V(x) = x(3 - 2x)^2$

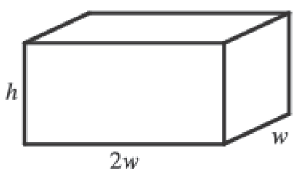
(f) $V(x) = x(3 - 2x)^2 \quad \Rightarrow$

$V'(x) = x \cdot 2(3 - 2x)(-2) + (3 - 2x)^2 \cdot 1 = (3 - 2x)[-4x + (3 - 2x)] = (3 - 2x)(-6x + 3)$,

so the critical numbers are $x = \frac{3}{2}$ and $x = \frac{1}{2}$. Now $0 \le x \le \frac{3}{2}$ and $V(0) = V\left(\frac{3}{2}\right) = 0$, so the maximum is

$V\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)(2)^2 = 2 \text{ ft}^3$, which is the value found from our third figure in part (a).

16.



$V = lwh \quad \Rightarrow \quad 10 = (2w)(w)h = 2w^2h$, so $h = 5/w^2$.

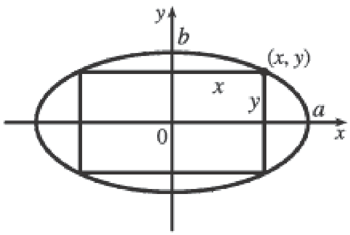
The cost is $10(2w^2) + 6[2(2wh) + 2(hw)] = 20w^2 + 36wh$, so

$C(w) = 20w^2 + 36w\left(5/w^2\right) = 20w^2 + 180/w$.

$C'(w) = 40w - 180/w^2 = 40\left(w^3 - \frac{9}{2}\right)/w^2 \quad \Rightarrow \quad w = \sqrt[3]{\frac{9}{2}}$ is the critical number. There is an absolute minimum for C when $w = \sqrt[3]{\frac{9}{2}}$ since $C'(w) < 0$ for $0 < w < \sqrt[3]{\frac{9}{2}}$ and $C'(w) > 0$ for $w > \sqrt[3]{\frac{9}{2}}$.

$C\left(\sqrt[3]{\frac{9}{2}}\right) = 20\left(\sqrt[3]{\frac{9}{2}}\right)^2 + \frac{180}{\sqrt[3]{9/2}} \approx \$163.54$.

24.



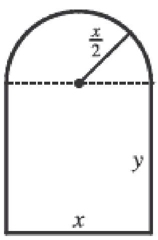
The area of the rectangle is $(2x)(2y) = 4xy$. Now $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ gives

$y = \dfrac{b}{a}\sqrt{a^2 - x^2}$, so we maximize $A(x) = 4\dfrac{b}{a}x\sqrt{a^2 - x^2}$.

$$A'(x) = \frac{4b}{a}\left[x \cdot \tfrac{1}{2}(a^2 - x^2)^{-1/2}(-2x) + (a^2 - x^2)^{1/2} \cdot 1\right]$$
$$= \frac{4b}{a}\,(a^2 - x^{2-1/2}[-x^2 + a^2 - x^2] = \frac{4b}{a\sqrt{a^2 - x^2}}[a^2 - 2x^2]$$

So the critical number is $x = \frac{1}{\sqrt{2}}\,a$, and this clearly gives a maximum. Then $y = \frac{1}{\sqrt{2}}\,b$, so the maximum area is $4\left(\frac{1}{\sqrt{2}}\,a\right)\left(\frac{1}{\sqrt{2}}\,b\right) = 2ab$.

32.



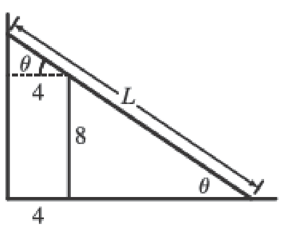
Perimeter $= 30 \quad \Rightarrow \quad 2y + x + \pi\left(\dfrac{x}{2}\right) = 30 \quad \Rightarrow$

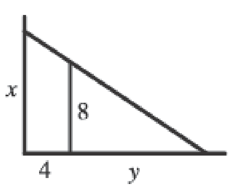
$y = \dfrac{1}{2}\left(30 - x - \dfrac{\pi x}{2}\right) = 15 - \dfrac{x}{2} - \dfrac{\pi x}{4}$. The area is the area of the rectangle plus the area of the semicircle, or $xy + \frac{1}{2}\pi\left(\dfrac{x}{2}\right)^2$, so $A(x) = x\left(15 - \dfrac{x}{2} - \dfrac{\pi x}{4}\right) + \frac{1}{8}\pi x^2 = 15x - \frac{1}{2}x^2 - \frac{\pi}{8}x^2$.

$A'(x) = 15 - \left(1 + \frac{\pi}{4}\right)x = 0 \quad \Rightarrow \quad x = \dfrac{15}{1 + \pi/4} = \dfrac{60}{4 + \pi}$. $\quad A''(x) = -\left(1 + \dfrac{\pi}{4}\right) < 0$, so this gives a maximum.

The dimensions are $x = \dfrac{60}{4+\pi}$ ft and $y = 15 - \dfrac{30}{4+\pi} - \dfrac{15\pi}{4+\pi} = \dfrac{60 + 15\pi - 30 - 15\pi}{4+\pi} = \dfrac{30}{4+\pi}$ ft, so the height of the rectangle is half the base.

38.





$L = 8\csc\theta + 4\sec\theta$, $0 < \theta < \frac{\pi}{2}$, $\dfrac{dL}{d\theta} = -8\csc\theta\,\cot\theta + 4\sec\theta\,\tan\theta = 0$ when

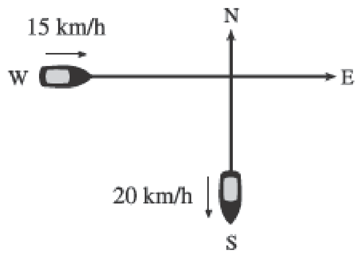
$\sec\theta\,\tan\theta = 2\csc\theta\,\cot\theta \quad \Leftrightarrow \quad \tan^3\theta = 2 \quad \Leftrightarrow \quad \tan\theta = \sqrt[3]{2} \quad \Leftrightarrow \quad \theta = \tan^{-1}\sqrt[3]{2}$.

$dL/d\theta < 0$ when $0 < \theta < \tan^{-1}\sqrt[3]{2}$, $dL/d\theta > 0$ when $\tan^{-1}\sqrt[3]{2} < \theta < \frac{\pi}{2}$, so L has an absolute minimum when $\theta = \tan^{-1}\sqrt[3]{2}$, and the shortest ladder has length

$$L = 8\frac{\sqrt{1 + 2^{2/3}}}{2^{1/3}} + 4\sqrt{1 + 2^{2/3}} \approx 16.65 \text{ ft.}$$

Another method: Minimize $L^2 = x^2 + (4 + y)^2$, where $\dfrac{x}{4+y} = \dfrac{8}{y}$.

46.

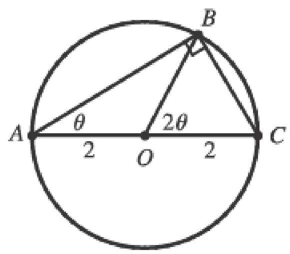


Let t be the time, in hours, after 2:00 PM. The position of the boat heading south at time t is $(0, -20t)$. The position of the boat heading east at time t is $(-15 + 15t, 0)$. If $D(t)$ is the distance between the boats at time t, we minimize $f(t) = [D(t)]^2 = 20^2t^2 + 15^2(t-1)^2$.

$f'(t) = 800t + 450(t-1) = 1250t - 450 = 0$ when $t = \frac{450}{1250} = 0.36$ h.

$0.36 \text{ h} \times \frac{60 \text{ min}}{\text{h}} = 21.6 \text{ min} = 21 \text{ min } 36 \text{ s}$. Since $f''(t) > 0$, this gives a minimum, so the boats are closest together at 2:21:36 PM.

48.



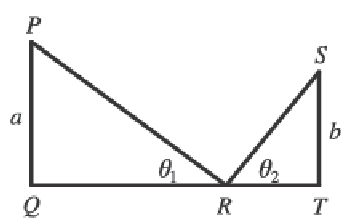
In isosceles triangle AOB, $\angle O = 180° - \theta - \theta$, so $\angle BOC = 2\theta$. The distance rowed is $4\cos\theta$ while the distance walked is the length of arc $BC = 2(2\theta) = 4\theta$. The time taken is given by $T(\theta) = \dfrac{4\cos\theta}{2} + \dfrac{4\theta}{4} = 2\cos\theta + \theta,\ 0 \le \theta \le \frac{\pi}{2}$.

$T'(\theta) = -2\sin\theta + 1 = 0 \quad\Leftrightarrow\quad \sin\theta = \frac{1}{2} \quad\Rightarrow\quad \theta = \frac{\pi}{6}$.

Check the value of T at $\theta = \frac{\pi}{6}$ and at the endpoints of the domain of T; that is, $\theta = 0$ and $\theta = \frac{\pi}{2}$.

$T(0) = 2$, $T\left(\frac{\pi}{6}\right) = \sqrt{3} + \frac{\pi}{6} \approx 2.26$, and $T\left(\frac{\pi}{2}\right) = \frac{\pi}{2} \approx 1.57$. Therefore, the minimum value of T is $\frac{\pi}{2}$ when $\theta = \frac{\pi}{2}$; that is, the woman should walk all the way. Note that $T''(\theta) = -2\cos\theta < 0$ for $0 \le \theta < \frac{\pi}{2}$, so $\theta = \frac{\pi}{6}$ gives a maximum time.

68.



If $d = |QT|$, we minimize $f(\theta_1) = |PR| + |RS| = a\csc\theta_1 + b\csc\theta_2$.

Differentiating with respect to θ_1, and setting $\dfrac{df}{d\theta_1}$ equal to 0, we get

$$\frac{df}{d\theta_1} = 0 = -a\csc\theta_1\ \cot\theta_1 - b\csc\theta_2\ \cot\theta_2\,\frac{d\theta_2}{d\theta_1}.$$

So we need to find an expression for $\dfrac{d\theta_2}{d\theta_1}$. We can do this by observing that $|QT| = \text{constant} = a\cot\theta_1 + b\cot\theta_2$.

Differentiating this equation implicitly with respect to θ_1, we get $-a\csc^2\theta_1 - b\csc^2\theta_2\,\dfrac{d\theta_2}{d\theta_1} = 0 \quad\Rightarrow$

$\dfrac{d\theta_2}{d\theta_1} = -\dfrac{a\csc^2\theta_1}{b\csc^2\theta_2}$. We substitute this into the expression for $\dfrac{df}{d\theta_1}$ to get

$$-a\csc\theta_1\ \cot\theta_1 - b\csc\theta_2\ \cot\theta_2\left(-\frac{a\csc^2\theta_1}{b\csc^2\theta_2}\right) = 0 \quad\Leftrightarrow\quad -a\csc\theta_1\ \cot\theta_1 + a\frac{\csc^2\theta_1\ \cot\theta_2}{\csc\theta_2} = 0 \quad\Leftrightarrow$$

$\cot\theta_1\ \csc\theta_2 = \csc\theta_1\ \cot\theta_2 \quad\Leftrightarrow\quad \dfrac{\cot\theta_1}{\csc\theta_1} = \dfrac{\cot\theta_2}{\csc\theta_2} \quad\Leftrightarrow\quad \cos\theta_1 = \cos\theta_2$. Since θ_1 and θ_2 are both acute, we have $\theta_1 = \theta_2$.

74. Let x be the distance from the observer to the wall. Then, from the given figure,

$$\theta = \tan^{-1}\left(\frac{h+d}{x}\right) - \tan^{-1}\left(\frac{d}{x}\right), \quad x > 0 \quad \Rightarrow$$

$$\frac{d\theta}{dx} = \frac{1}{1+[(h+d)/x]^2}\left[-\frac{h+d}{x^2}\right] - \frac{1}{1+(d/x)^2}\left[-\frac{d}{x^2}\right] = -\frac{h+d}{x^2+(h+d)^2} + \frac{d}{x^2+d^2}$$

$$= \frac{d[x^2+(h+d)^2] - (h+d)(x^2+d^2)}{[x^2+(h+d)^2](x^2+d^2)} = \frac{h^2d + hd^2 - hx^2}{[x^2+(h+d)^2](x^2+d^2)} = 0 \quad \Leftrightarrow$$

$hx^2 = h^2d + hd^2 \quad \Leftrightarrow \quad x^2 = hd + d^2 \quad \Leftrightarrow \quad x = \sqrt{d(h+d)}$. Since $d\theta/dx > 0$ for all $x < \sqrt{d(h+d)}$ and $d\theta/dx < 0$ for all $x > \sqrt{d(h+d)}$, the absolute maximum occurs when $x = \sqrt{d(h+d)}$.