6. (a) $\sinh 1 = \frac{1}{2}(e^1 - e^{-1}) \approx 1.17520$

(b) Using Equation 3, we have $\sinh^{-1} 1 = \ln\left(1 + \sqrt{1^2 + 1}\right) = \ln\left(1 + \sqrt{2}\right) \approx 0.88137$.

8. $\cosh(-x) = \frac{1}{2}[e^{-x} + e^{-(-x)}] = \frac{1}{2}(e^{-x} + e^{x}) = \frac{1}{2}(e^{x} + e^{-x}) = \cosh x$

20. $\coth x = \dfrac{1}{\tanh x} \quad \Rightarrow \quad \coth x = \dfrac{1}{\tanh x} = \dfrac{1}{12/13} = \dfrac{13}{12}$.

$\operatorname{sech}^2 x = 1 - \tanh^2 x = 1 - \left(\frac{12}{13}\right)^2 = \frac{25}{169} \quad \Rightarrow \quad \operatorname{sech} x = \frac{5}{13}$ [sech, like cosh, is positive].

$\cosh x = \dfrac{1}{\operatorname{sech} x} \quad \Rightarrow \quad \cosh x = \dfrac{1}{5/13} = \dfrac{13}{5}$.

$\tanh x = \dfrac{\sinh x}{\cosh x} \quad \Rightarrow \quad \sinh x = \tanh x \, \cosh x \quad \Rightarrow \quad \sinh x = \dfrac{12}{13} \cdot \dfrac{13}{5} = \dfrac{12}{5}$.

$\operatorname{csch} x = \dfrac{1}{\sinh x} \quad \Rightarrow \quad \operatorname{csch} x = \dfrac{1}{12/5} = \dfrac{5}{12}$.

30. $f(x) = \tanh(1 + e^{2x}) \quad \Rightarrow \quad f'(x) = \operatorname{sech}^2(1 + e^{2x}) \dfrac{d}{dx}(1 + e^{2x}) = 2e^{2x} \operatorname{sech}^2(1 + e^{2x})$

34. $y = x \coth(1 + x^2) \quad \Rightarrow \quad y' = x[-\operatorname{csch}^2(1 + x^2) \cdot 2x] + \coth(1 + x^2) \cdot 1 = -2x^2 \operatorname{csch}^2(1 + x^2) + \coth(1 + x^2)$

42. $y = x \tanh^{-1} x + \ln\sqrt{1 - x^2} = x \tanh^{-1} x + \frac{1}{2}\ln(1 - x^2) \quad \Rightarrow$

$y' = \tanh^{-1} x + \dfrac{x}{1 - x^2} + \dfrac{1}{2}\left(\dfrac{1}{1 - x^2}\right)(-2x) = \tanh^{-1} x$

52. We differentiate the function twice, then substitute into the differential equation: $y = \dfrac{T}{\rho g} \cosh \dfrac{\rho g x}{T} \quad \Rightarrow$

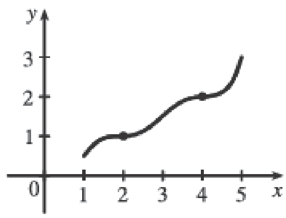
$\dfrac{dy}{dx} = \dfrac{T}{\rho g} \sinh\left(\dfrac{\rho g x}{T}\right) \dfrac{\rho g}{T} = \sinh \dfrac{\rho g x}{T} \quad \Rightarrow \quad \dfrac{d^2 y}{dx^2} = \cosh\left(\dfrac{\rho g x}{T}\right) \dfrac{\rho g}{T} = \dfrac{\rho g}{T} \cosh \dfrac{\rho g x}{T}$. We evaluate the two sides separately: LHS $= \dfrac{d^2 y}{dx^2} = \dfrac{\rho g}{T} \cosh \dfrac{\rho g x}{T}$ and RHS $= \dfrac{\rho g}{T} \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2} = \dfrac{\rho g}{T} \sqrt{1 + \sinh^2 \dfrac{\rho g x}{T}} = \dfrac{\rho g}{T} \cosh \dfrac{\rho g x}{T}$,

by the identity proved in Example 1(a).

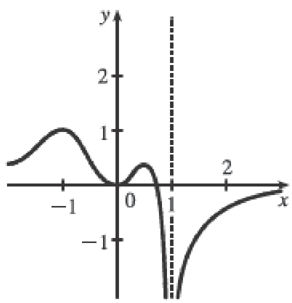
4. Absolute maximum at r; absolute minimum at a; local maxima at b and r; local minimum at d; neither a maximum nor a minimum at c and s.

6. There is no absolute maximum value; absolute minimum value is $g(4) = 1$; local maximum values are $g(3) = 4$ and $g(6) = 3$; local minimum values are $g(2) = 2$ and $g(4) = 1$.

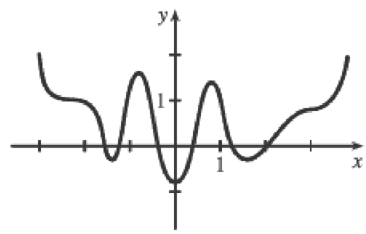
10. f has no local maximum or minimum, but 2 and 4 are critical numbers



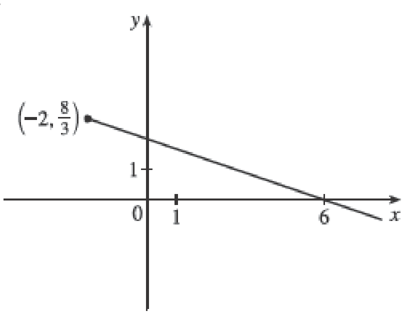
14. (a)



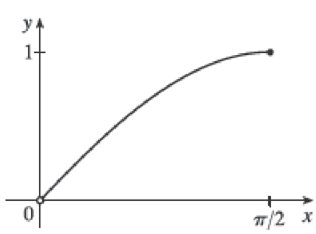
(b)



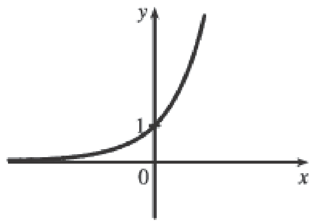
16. $f(x) = 2 - \frac{1}{3}x$, $x \geq -2$. Absolute maximum $f(-2) = \frac{8}{3}$; no local maximum. No absolute or local minimum.



20. $f(x) = \sin x$, $0 < x \leq \pi/2$. Absolute maximum $f\left(\frac{\pi}{2}\right) = 1$; no local maximum. No absolute or local minimum.



26. $f(x) = e^x$. No absolute or local maximum or minimum value.



32. $f(x) = 2x^3 + x^2 + 2x \quad \Rightarrow \quad f'(x) = 6x^2 + 2x + 2 = 2(3x^2 + x + 1)$. Using the quadratic formula, $f'(x) = 0 \quad \Leftrightarrow$ $x = \dfrac{-1 \pm \sqrt{-11}}{6}$. Since the discrimininant, -11, is negative, there are no real soutions, and hence, there are no critical numbers.

34. $g(t) = |3t - 4| = \begin{cases} 3t - 4 & \text{if } 3t - 4 \geq 0 \\ -(3t - 4) & \text{if } 3t - 4 < 0 \end{cases} = \begin{cases} 3t - 4 & \text{if } t \geq \frac{4}{3} \\ 4 - 3t & \text{if } t < \frac{4}{3} \end{cases}$

$g'(t) = \begin{cases} 3 & \text{if } t > \frac{4}{3} \\ -3 & \text{if } t < \frac{4}{3} \end{cases}$ and $g'(t)$ does not exist at $t = \frac{4}{3}$, so $t = \frac{4}{3}$ is a critical number.

38. $g(x) = x^{1/3} - x^{-2/3} \quad \Rightarrow \quad g'(x) = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-5/3} = \frac{1}{3}x^{-5/3}(x + 2) = \dfrac{x + 2}{3x^{5/3}}$.

$g'(-2) = 0$ and $g'(0)$ does not exist, but 0 is not in the domain of g, so the only critical number is -2.

40. $g(\theta) = 4\theta - \tan\theta \quad \Rightarrow \quad g'(\theta) = 4 - \sec^2\theta$. $g'(\theta) = 0 \quad \Rightarrow \quad \sec^2\theta = 4 \quad \Rightarrow \quad \sec\theta = \pm 2 \quad \Rightarrow \quad \cos\theta = \pm\frac{1}{2} \quad \Rightarrow$

$\theta = \frac{\pi}{3} + 2n\pi$, $\frac{5\pi}{3} + 2n\pi$, $\frac{2\pi}{3} + 2n\pi$, and $\frac{4\pi}{3} + 2n\pi$ are critical numbers.

Note: The values of θ that make $g'(\theta)$ undefined are not in the domain of g.

48. $f(x) = 5 + 54x - 2x^3$, $[0, 4]$. $f'(x) = 54 - 6x^2 = 6(9 - x^2) = 6(3 + x)(3 - x) = 0 \quad \Leftrightarrow \quad x = -3, 3$. $f(0) = 5$, $f(3) = 113$, and $f(4) = 93$. So $f(3) = 113$ is the absolute maximum value and $f(0) = 5$ is the absolute minimum value.

54. $f(x) = \dfrac{x}{x^2 - x + 1}$, $[0, 3]$.

$f'(x) = \dfrac{(x^2 - x + 1) - x(2x - 1)}{(x^2 - x + 1)^2} = \dfrac{x^2 - x + 1 - 2x^2 + x}{(x^2 - x + 1)^2} = \dfrac{1 - x^2}{(x^2 - x + 1)^2} = \dfrac{(1 + x)(1 - x)}{(x^2 - x + 1)^2} = 0 \quad \Leftrightarrow \quad x = \pm 1$,

but $x = -1$ is not in the given interval, $[0, 3]$. $f(0) = 0$, $f(1) = 1$, and $f(3) = \frac{3}{7}$. So $f(1) = 1$ is the absolute maximum value and $f(0) = 0$ is the absolute minimum value.

60. $f(x) = x - \ln x$, $[\frac{1}{2}, 2]$. $f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x}$. $f'(x) = 0 \Rightarrow x = 1$. [Note that 0 is not in the domain of f.]
$f\left(\frac{1}{2}\right) = \frac{1}{2} - \ln \frac{1}{2} \approx 1.19$, $f(1) = 1$, and $f(2) = 2 - \ln 2 \approx 1.31$. So $f(2) = 2 - \ln 2$ is the absolute maximum value and $f(1) = 1$ is the absolute minimum value.