4. By the Product Rule, $g(x) = \sqrt{x}\,e^x = x^{1/2}e^x \quad \Rightarrow \quad g'(x) = x^{1/2}(e^x) + e^x\left(\frac{1}{2}x^{-1/2}\right) = \frac{1}{2}x^{-1/2}e^x(2x+1)$.

6. By the Quotient Rule, $y = \dfrac{e^x}{1-e^x} \quad \Rightarrow \quad y' = \dfrac{(1-e^x)e^x - e^x(-e^x)}{(1-e^x)^2} = \dfrac{e^x - e^{2x} + e^{2x}}{(1-e^x)^2} = \dfrac{e^x}{(1-e^x)^2}$.

10. $J(v) = (v^3 - 2v)(v^{-4} + v^{-2}) \quad \overset{\text{PR}}{\Rightarrow}$

$$J'(v) = (v^3 - 2v)(-4v^{-5} - 2v^{-3}) + (v^{-4} + v^{-2})(3v^2 - 2)$$
$$= -4v^{-2} - 2v^0 + 8v^{-4} + 4v^{-2} + 3v^{-2} - 2v^{-4} + 3v^0 - 2v^{-2} = 1 + v^{-2} + 6v^{-4}$$

14. $y = \dfrac{x+1}{x^3+x-2} \quad \overset{\text{QR}}{\Rightarrow}$

$$y' = \frac{(x^3+x-2)(1) - (x+1)(3x^2+1)}{(x^3+x-2)^2} = \frac{x^3+x-2-3x^3-3x^2-x-1}{(x^3+x-2)^2} = \frac{-2x^3-3x^2-3}{(x^3+x-2)^2}$$

or $-\dfrac{2x^3+3x^2+3}{(x-1)^2(x^2+x+2)^2}$

22. $g(t) = \dfrac{t - \sqrt{t}}{t^{1/3}} = \dfrac{t}{t^{1/3}} - \dfrac{t^{1/2}}{t^{1/3}} = t^{2/3} - t^{1/6} \quad \Rightarrow \quad g'(t) = \frac{2}{3}t^{-1/3} - \frac{1}{6}t^{-5/6}$

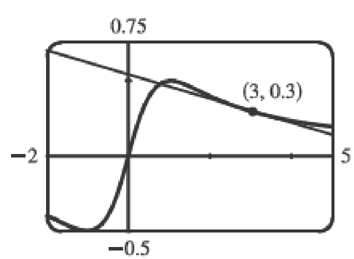
30. $f(x) = \dfrac{x}{x^2-1} \quad \Rightarrow \quad f'(x) = \dfrac{(x^2-1)(1) - x(2x)}{(x^2-1)^2} = \dfrac{x^2-1-2x^2}{(x^2-1)^2} = \dfrac{-x^2-1}{(x^2-1)^2} \quad \Rightarrow$

$$f''(x) = \frac{(x^2-1)^2(-2x) - (-x^2-1)(x^4-2x^2+1)'}{[(x^2-1)^2]^2} = \frac{(x^2-1)^2(-2x) + (x^2+1)(4x^3-4x)}{(x^2-1)^4}$$
$$= \frac{(x^2-1)^2(-2x) + (x^2+1)(4x)(x^2-1)}{(x^2-1)^4} = \frac{(x^2-1)[(x^2-1)(-2x) + (x^2+1)(4x)]}{(x^2-1)^4}$$
$$= \frac{-2x^3+2x+4x^3+4x}{(x^2-1)^3} = \frac{2x^3+6x}{(x^2-1)^3}$$

36. (a) $y = f(x) = \dfrac{x}{1+x^2} \quad \Rightarrow$

$f'(x) = \dfrac{(1+x^2)1 - x(2x)}{(1+x^2)^2} = \dfrac{1-x^2}{(1+x^2)^2}$. So the slope of the tangent line at the point $(3, 0.3)$ is $f'(3) = \frac{-8}{100}$ and its equation is $y - 0.3 = -0.08(x-3)$ or $y = -0.08x + 0.54$.

(b)



42. $g(x) = \frac{x}{e^x} \quad \Rightarrow \quad g'(x) = \frac{e^x \cdot 1 - x \cdot e^x}{(e^x)^2} = \frac{e^x(1-x)}{(e^x)^2} = \frac{1-x}{e^x} \quad \Rightarrow$

$g''(x) = \frac{e^x \cdot (-1) - (1-x)e^x}{(e^x)^2} = \frac{e^x[-1-(1-x)]}{(e^x)^2} = \frac{x-2}{e^x} \quad \Rightarrow$

$g'''(x) = \frac{e^x \cdot 1 - (x-2)e^x}{(e^x)^2} = \frac{e^x[1-(x-2)]}{(e^x)^2} = \frac{3-x}{e^x} \quad \Rightarrow$

$g^{(4)}(x) = \frac{e^x \cdot (-1) - (3-x)e^x}{(e^x)^2} = \frac{e^x[-1-(3-x)]}{(e^x)^2} = \frac{x-4}{e^x}.$

The pattern suggests that $g^{(n)}(x) = \frac{(x-n)(-1)^n}{e^x}$. (We could use mathematical induction to prove this formula.)

46. $\frac{d}{dx}\left[\frac{h(x)}{x}\right] = \frac{xh'(x) - h(x) \cdot 1}{x^2} \quad \Rightarrow \quad \frac{d}{dx}\left[\frac{h(x)}{x}\right]_{x=2} = \frac{2h'(2) - h(2)}{2^2} = \frac{2(-3) - (4)}{4} = \frac{-10}{4} = -2.5$

50. (a) $P(x) = F(x)\,G(x)$, so $P'(2) = F(2)\,G'(2) + G(2)\,F'(2) = 3 \cdot \frac{2}{4} + 2 \cdot 0 = \frac{3}{2}$.

(b) $Q(x) = F(x)/G(x)$, so $Q'(7) = \frac{G(7)\,F'(7) - F(7)\,G'(7)}{[G(7)]^2} = \frac{1 \cdot \frac{1}{4} - 5 \cdot \left(-\frac{2}{3}\right)}{1^2} = \frac{1}{4} + \frac{10}{3} = \frac{43}{12}$

2. $f(x) = \sqrt{x}\sin x \quad \Rightarrow \quad f'(x) = \sqrt{x}\cos x + \sin x\left(\frac{1}{2}x^{-1/2}\right) = \sqrt{x}\cos x + \frac{\sin x}{2\sqrt{x}}$

8. $f(t) = \frac{\cot t}{e^t} \quad \Rightarrow \quad f'(t) = \frac{e^t(-\csc^2 t) - (\cot t)e^t}{(e^t)^2} = \frac{e^t(-\csc^2 t - \cot t)}{(e^t)^2} = -\frac{\csc^2 t + \cot t}{e^t}$

10. $y = \sin\theta\,\cos\theta \quad \Rightarrow \quad y' = \sin\theta(-\sin\theta) + \cos\theta(\cos\theta) = \cos^2\theta - \sin^2\theta \quad$ [or $\cos 2\theta$]

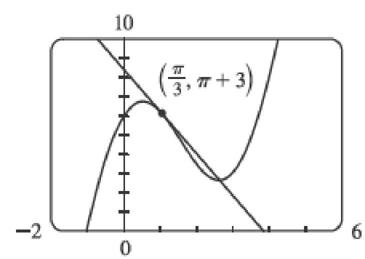
24. $y = x + \tan x \quad \Rightarrow \quad y' = 1 + \sec^2 x$, so $y'(\pi) = 1 + (-1)^2 = 2$. An equation of the tangent line to the curve $y = x + \tan x$ at the point (π, π) is $y - \pi = 2(x - \pi)$ or $y = 2x - \pi$.

26. (a) $y = 3x + 6\cos x \quad \Rightarrow \quad y' = 3 - 6\sin x$. At $\left(\frac{\pi}{3}, \pi + 3\right)$,

$y' = 3 - 6\sin\frac{\pi}{3} = 3 - 6\,\frac{\sqrt{3}}{2} = 3 - 3\sqrt{3}$, and an equation of the tangent line is $y - (\pi + 3) = \left(3 - 3\sqrt{3}\right)\left(x - \frac{\pi}{3}\right)$, or

$y = \left(3 - 3\sqrt{3}\right)x + 3 + \pi\sqrt{3}$.

(b)



32. (a) $g(x) = f(x)\sin x \quad \Rightarrow \quad g'(x) = f(x)\cos x + \sin x \cdot f'(x)$, so

$$g'(\tfrac{\pi}{3}) = f(\tfrac{\pi}{3})\cos\tfrac{\pi}{3} + \sin\tfrac{\pi}{3}\cdot f'(\tfrac{\pi}{3}) = 4\cdot\tfrac{1}{2} + \tfrac{\sqrt{3}}{2}\cdot(-2) = 2-\sqrt{3}$$

(b) $h(x) = \dfrac{\cos x}{f(x)} \quad \Rightarrow \quad h'(x) = \dfrac{f(x)\cdot(-\sin x) - \cos x\cdot f'(x)}{[f(x)]^2}$, so

$$h'(\tfrac{\pi}{3}) = \frac{f(\frac{\pi}{3})\cdot(-\sin\frac{\pi}{3}) - \cos\frac{\pi}{3}\cdot f'(\frac{\pi}{3})}{\left[f(\frac{\pi}{3})\right]^2} = \frac{4\left(-\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)(-2)}{4^2} = \frac{-2\sqrt{3}+1}{16} = \frac{1-2\sqrt{3}}{16}$$

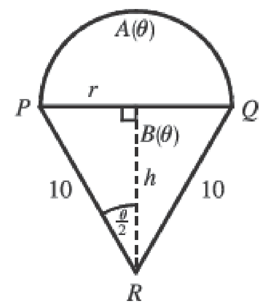
44.
$$\lim_{x\to 0}\frac{\sin 3x\sin 5x}{x^2} = \lim_{x\to 0}\left(\frac{3\sin 3x}{3x}\cdot\frac{5\sin 5x}{5x}\right) = \lim_{x\to 0}\frac{3\sin 3x}{3x}\cdot\lim_{x\to 0}\frac{5\sin 5x}{5x}$$
$$= 3\lim_{x\to 0}\frac{\sin 3x}{3x}\cdot 5\lim_{x\to 0}\frac{\sin 5x}{5x} = 3(1)\cdot 5(1) = 15$$

54. We get the following formulas for r and h in terms of θ:

$$\sin\frac{\theta}{2} = \frac{r}{10} \quad\Rightarrow\quad r = 10\sin\frac{\theta}{2} \quad\text{and}\quad \cos\frac{\theta}{2} = \frac{h}{10} \quad\Rightarrow\quad h = 10\cos\frac{\theta}{2}$$

Now $A(\theta) = \frac{1}{2}\pi r^2$ and $B(\theta) = \frac{1}{2}(2r)h = rh$. So

$$\lim_{\theta\to 0^+}\frac{A(\theta)}{B(\theta)} = \lim_{\theta\to 0^+}\frac{\frac{1}{2}\pi r^2}{rh} = \tfrac{1}{2}\pi\lim_{\theta\to 0^+}\frac{r}{h} = \tfrac{1}{2}\pi\lim_{\theta\to 0^+}\frac{10\sin(\theta/2)}{10\cos(\theta/2)}$$
$$= \tfrac{1}{2}\pi\lim_{\theta\to 0^+}\tan(\theta/2) = 0$$



6. Let $u = g(x) = 2 - e^x$ and $y = f(u) = \sqrt{u}$. Then $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx} = (\frac{1}{2}u^{-1/2})(-e^x) = -\dfrac{e^x}{2\sqrt{2-e^x}}$.

12. $f(t) = \sin(e^t) + e^{\sin t} \quad\Rightarrow\quad f'(t) = \cos(e^t)\cdot e^t + e^{\sin t}\cdot\cos t = e^t\cos(e^t) + e^{\sin t}\cos t$

16. $y = e^{-2t}\cos 4t \quad\Rightarrow\quad y' = e^{-2t}(-\sin 4t\cdot 4) + \cos 4t[e^{-2t}(-2)] = -2e^{-2t}(2\sin 4t + \cos 4t)$

22. $f(s) = \sqrt{\dfrac{s^2+1}{s^2+4}} \quad\Rightarrow$

$$f'(s) = \frac{1}{2}\left(\frac{s^2+1}{s^2+4}\right)^{-1/2}\frac{(s^2+4)(2s)-(s^2+1)(2s)}{(s^2+4)^2} = \frac{1}{2}\left(\frac{s^2+4}{s^2+1}\right)^{1/2}\frac{2s[(s^2+4)-(s^2+1)]}{(s^2+4)^2}$$
$$= \frac{(s^2+4)^{1/2}(2s)(3)}{2(s^2+1)^{1/2}(s^2+4)^2} = \frac{3s}{(s^2+1)^{1/2}(s^2+4)^{3/2}}$$

26. $G(y) = \dfrac{(y-1)^4}{(y^2+2y)^5} \quad \Rightarrow$

$$G'(y) = \frac{(y^2+2y)^5 \cdot 4(y-1)^3 \cdot 1 - (y-1)^4 \cdot 5(y^2+2y)^4(2y+2)}{\left[(y^2+2y)^5\right]^2}$$

$$= \frac{2(y^2+2y)^4(y-1)^3\left[2(y^2+2y) - 5(y-1)(y+1)\right]}{(y^2+2y)^{10}}$$

$$= \frac{2(y-1)^3\left[(2y^2+4y)+(-5y^2+5)\right]}{(y^2+2y)^6} = \frac{2(y-1)^3(-3y^2+4y+5)}{(y^2+2y)^6}$$

30. $F(v) = \left(\dfrac{v}{v^3+1}\right)^6 \quad \Rightarrow$

$$F'(v) = 6\left(\frac{v}{v^3+1}\right)^5 \frac{(v^3+1)(1) - v(3v^2)}{(v^3+1)^2} = \frac{6v^5(v^3+1-3v^3)}{(v^3+1)^5(v^3+1)^2} = \frac{6v^5(1-2v^3)}{(v^3+1)^7}$$

36. $y = \sqrt{1+xe^{-2x}} \quad \Rightarrow \quad y' = \frac{1}{2}\left(1+xe^{-2x}\right)^{-1/2}\left[x\left(-2e^{-2x}\right)+e^{-2x}\right] = \dfrac{e^{-2x}(-2x+1)}{2\sqrt{1+xe^{-2x}}}$

44. $y = 2^{3^{x^2}} \quad \Rightarrow \quad y' = 2^{3^{x^2}}(\ln 2)\dfrac{d}{dx}\left(3^{x^2}\right) = 2^{3^{x^2}}(\ln 2)3^{x^2}(\ln 3)(2x)$

48. $y = \cos^2 x = (\cos x)^2 \quad \Rightarrow \quad y' = 2\cos x(-\sin x) = -2\cos x \sin x \quad \Rightarrow$

$y'' = (-2\cos x)\cos x + \sin x(2\sin x) = -2\cos^2 x + 2\sin^2 x$

Note: Many other forms of the answers exist. For example, $y' = -\sin 2x$ and $y'' = -2\cos 2x$.

54. $y = \sin x + \sin^2 x \quad \Rightarrow \quad y' = \cos x + 2\sin x\, \cos x.$

At $(0, 0)$, $y' = 1$, and an equation of the tangent line is $y - 0 = 1(x-0)$, or $y = x$.

66. (a) $h(x) = f(f(x)) \quad \Rightarrow \quad h'(x) = f'(f(x))f'(x)$. So $h'(2) = f'(f(2))f'(2) = f'(1)f'(2) \approx (-1)(-1) = 1$.

(b) $g(x) = f(x^2) \quad \Rightarrow \quad g'(x) = f'(x^2)\cdot \dfrac{d}{dx}\left(x^2\right) = f'(x^2)(2x)$. So $g'(2) = f'(2^2)(2\cdot 2) = 4f'(4) \approx 4(2) = 8$.

72. $f(x) = xg(x^2) \quad \Rightarrow \quad f'(x) = xg'(x^2)\,2x + g(x^2)\cdot 1 = 2x^2 g'(x^2) + g(x^2) \quad \Rightarrow$

$f''(x) = 2x^2 g''(x^2)\,2x + g'(x^2)\,4x + g'(x^2)\,2x = 4x^3 g''(x^2) + 4xg'(x^2) + 2xg'(x^2) = 6xg'(x^2) + 4x^3 g''(x^2)$