**4.** (a) (i) Using Definition 1 with $f(x) = x - x^3$ and $P(1, 0)$,

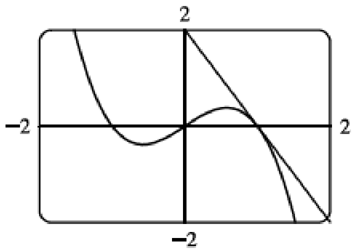
$$m = \lim_{x\to 1} \frac{f(x) - 0}{x - 1} = \lim_{x\to 1} \frac{x - x^3}{x - 1} = \lim_{x\to 1} \frac{x(1 - x^2)}{x - 1} = \lim_{x\to 1} \frac{x(1 + x)(1 - x)}{x - 1}$$
$$= \lim_{x\to 1} [-x(1 + x)] = -1(2) = -2$$

(ii) Using Equation 2 with $f(x) = x - x^3$ and $P(1, 0)$,

$$m = \lim_{h\to 0} \frac{f(a + h) - f(a)}{h} = \lim_{h\to 0} \frac{f(1 + h) - f(1)}{h} = \lim_{h\to 0} \frac{\left[(1 + h) - (1 + h)^3\right] - 0}{h}$$
$$= \lim_{h\to 0} \frac{1 + h - (1 + 3h + 3h^2 + h^3)}{h} = \lim_{h\to 0} \frac{-h^3 - 3h^2 - 2h}{h} = \lim_{h\to 0} \frac{h(-h^2 - 3h - 2)}{h}$$
$$= \lim_{h\to 0} (-h^2 - 3h - 2) = -2$$

(b) An equation of the tangent line is $y - f(a) = f'(a)(x - a) \;\Rightarrow\; y - f(1) = f'(1)(x - 1) \;\Rightarrow\; y - 0 = -2(x - 1)$, or $y = -2x + 2$.

(c)




The graph of $y = -2x + 2$ is tangent to the graph of $y = x - x^3$ at the point $(1, 0)$. Now zoom in toward the point $(1, 0)$ until the cubic and the tangent line are indistinguishable.

**6.** Using (2) with $f(x) = x^3 - 3x + 1$ and $P(2, 3)$,

$$m = \lim_{h\to 0} \frac{f(a + h) - f(a)}{h} = \lim_{h\to 0} \frac{f(2 + h) - f(2)}{h} = \lim_{h\to 0} \frac{(2 + h)^3 - 3(2 + h) + 1 - 3}{h}$$
$$= \lim_{h\to 0} \frac{8 + 12h + 6h^2 + h^3 - 6 - 3h - 2}{h} = \lim_{h\to 0} \frac{9h + 6h^2 + h^3}{h} = \lim_{h\to 0} \frac{h(9 + 6h + h^2)}{h}$$
$$= \lim_{h\to 0} (9 + 6h + h^2) = 9$$

Tangent line: $y - 3 = 9(x - 2) \;\Leftrightarrow\; y - 3 = 9x - 18 \;\Leftrightarrow\; y = 9x - 15$

**10.** (a) Using (1),

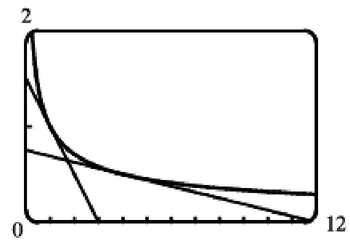
$$m = \lim_{x \to a} \frac{\dfrac{1}{\sqrt{x}} - \dfrac{1}{\sqrt{a}}}{x - a} = \lim_{x \to a} \frac{\dfrac{\sqrt{a} - \sqrt{x}}{\sqrt{ax}}}{x - a} = \lim_{x \to a} \frac{(\sqrt{a} - \sqrt{x})(\sqrt{a} + \sqrt{x})}{\sqrt{ax}\,(x - a)(\sqrt{a} + \sqrt{x})} = \lim_{x \to a} \frac{a - x}{\sqrt{ax}\,(x - a)(\sqrt{a} + \sqrt{x})}$$

$$= \lim_{x \to a} \frac{-1}{\sqrt{ax}\,(\sqrt{a} + \sqrt{x})} = \frac{-1}{\sqrt{a^2}\,(2\sqrt{a})} = -\frac{1}{2a^{3/2}} \text{ or } -\frac{1}{2}a^{-3/2} \ [a > 0]$$

(b) At $(1, 1)$: $m = -\frac{1}{2}$, so an equation of the tangent line is $y - 1 = -\frac{1}{2}(x - 1) \quad \Leftrightarrow \quad y = -\frac{1}{2}x + \frac{3}{2}$.

At $(4, \frac{1}{2})$: $m = -\frac{1}{16}$, so an equation of the tangent line is $y - \frac{1}{2} = -\frac{1}{16}(x - 4) \quad \Leftrightarrow \quad y = -\frac{1}{16}x + \frac{3}{4}$.

(c)




**14.** (a) Let $H(t) = 10t - 1.86t^2$.

$$v(1) = \lim_{h \to 0} \frac{H(1 + h) - H(1)}{h} = \lim_{h \to 0} \frac{[10(1 + h) - 1.86(1 + h)^2] - (10 - 1.86)}{h}$$

$$= \lim_{h \to 0} \frac{10 + 10h - 1.86(1 + 2h + h^2) - 10 + 1.86}{h}$$

$$= \lim_{h \to 0} \frac{10 + 10h - 1.86 - 3.72h - 1.86h^2 - 10 + 1.86}{h}$$

$$= \lim_{h \to 0} \frac{6.28h - 1.86h^2}{h} = \lim_{h \to 0} (6.28 - 1.86h) = 6.28$$

The velocity of the rock after one second is 6.28 m/s.

(b) $$v(a) = \lim_{h \to 0} \frac{H(a + h) - H(a)}{h} = \lim_{h \to 0} \frac{[10(a + h) - 1.86(a + h)^2] - (10a - 1.86a^2)}{h}$$

$$= \lim_{h \to 0} \frac{10a + 10h - 1.86(a^2 + 2ah + h^2) - 10a + 1.86a^2}{h}$$

$$= \lim_{h \to 0} \frac{10a + 10h - 1.86a^2 - 3.72ah - 1.86h^2 - 10a + 1.86a^2}{h} = \lim_{h \to 0} \frac{10h - 3.72ah - 1.86h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(10 - 3.72a - 1.86h)}{h} = \lim_{h \to 0} (10 - 3.72a - 1.86h) = 10 - 3.72a$$

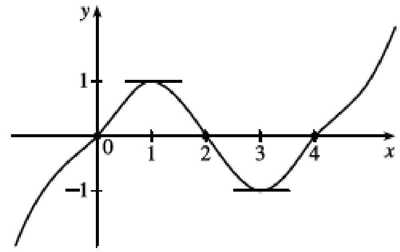
The velocity of the rock when $t = a$ is $(10 - 3.72a)$ m/s.

(c) The rock will hit the surface when $H = 0 \quad \Leftrightarrow \quad 10t - 1.86t^2 = 0 \quad \Leftrightarrow \quad t(10 - 1.86t) = 0 \quad \Leftrightarrow \quad t = 0$ or $1.86t = 10$.

The rock hits the surface when $t = 10/1.86 \approx 5.4$ s.

(d) The velocity of the rock when it hits the surface is $v\left(\frac{10}{1.86}\right) = 10 - 3.72\left(\frac{10}{1.86}\right) = 10 - 20 = -10$ m/s.

**22.** We begin by drawing a curve through the origin with a slope of 1 to satisfy $g(0) = 0$ and $g'(0) = 1$. We round off our figure at $x = 1$ to satisfy $g'(1) = 0$, and then pass through $(2, 0)$ with slope $-1$ to satisfy $g(2) = 0$ and $g'(2) = -1$. We round the figure at $x = 3$ to satisfy $g'(3) = 0$, and then pass through $(4, 0)$ with slope 1 to satisfy $g(4) = 0$ and $g'(4) = 1$. Finally we extend the curve on both ends to satisfy $\lim_{x\to\infty} g(x) = \infty$ and $\lim_{x\to-\infty} g(x) = -\infty$.




**30.** Use (4) with $f(x) = x^{-2} = 1/x^2$.

$$f'(a) = \lim_{h\to0}\frac{f(a+h)-f(a)}{h} = \lim_{h\to0}\frac{\frac{1}{(a+h)^2}-\frac{1}{a^2}}{h} = \lim_{h\to0}\frac{\frac{a^2-(a+h)^2}{a^2(a+h)^2}}{h} = \lim_{h\to0}\frac{a^2-(a^2+2ah+h^2)}{ha^2(a+h)^2}$$

$$= \lim_{h\to0}\frac{-2ah-h^2}{ha^2(a+h)^2} = \lim_{h\to0}\frac{h(-2a-h)}{ha^2(a+h)^2} = \lim_{h\to0}\frac{-2a-h}{a^2(a+h)^2} = \frac{-2a}{a^2(a^2)} = \frac{-2}{a^3}$$

Note that the answers to Exercises 33 – 38 are not unique.

**36.** By Equation 5, $\lim_{x\to\pi/4}\frac{\tan x - 1}{x - \pi/4} = f'(\pi/4)$, where $f(x) = \tan x$ and $a = \pi/4$.

**40.** $v(5) = f'(5) = \lim_{h\to0}\frac{f(5+h)-f(5)}{h} = \lim_{h\to0}\frac{[(5+h)^{-1}-(5+h)]-(5^{-1}-5)}{h}$

$$= \lim_{h\to0}\frac{\frac{1}{5+h}-5-h-\frac{1}{5}+5}{h} = \lim_{h\to0}\frac{\frac{1}{5+h}-h-\frac{1}{5}}{h} = \lim_{h\to0}\frac{\frac{5-5h(5+h)-(5+h)}{5(5+h)}}{h}$$

$$= \lim_{h\to0}\frac{5-25h-5h^2-5-h}{5h(5+h)} = \lim_{h\to0}\frac{-5h^2-26h}{5h(5+h)} = \lim_{h\to0}\frac{h(-5h-26)}{5h(5+h)} = \lim_{h\to0}\frac{-5h-26}{5(5+h)} = \frac{-26}{25}\text{ m/s}$$

The speed when $t = 5$ is $\left|-\frac{26}{25}\right| = \frac{26}{25} = 1.04$ m/s.

**46.** $\Delta V = V(t+h) - V(t) = 100{,}000\left(1 - \frac{t+h}{60}\right)^2 - 100{,}000\left(1 - \frac{t}{60}\right)^2$

$$= 100{,}000\left[\left(1 - \frac{t+h}{30} + \frac{(t+h)^2}{3600}\right) - \left(1 - \frac{t}{30} + \frac{t^2}{3600}\right)\right] = 100{,}000\left(-\frac{h}{30} + \frac{2th}{3600} + \frac{h^2}{3600}\right)$$

$$= \frac{100{,}000}{3600}h(-120 + 2t + h) = \frac{250}{9}h(-120 + 2t + h)$$

Dividing $\Delta V$ by $h$ and then letting $h \to 0$, we see that the instantaneous rate of change is $\frac{500}{9}(t - 60)$ gal/min.

| $t$ | Flow rate (gal/min) | Water remaining $V(t)$ (gal) |
|---|---|---|
| 0 | $-3333.\overline{3}$ | $100,000$ |
| 10 | $-2777.\overline{7}$ | $69,444.\overline{4}$ |
| 20 | $-2222.\overline{2}$ | $44,444.\overline{4}$ |
| 30 | $-1666.\overline{6}$ | $25,000$ |
| 40 | $-1111.\overline{1}$ | $11,111.\overline{1}$ |
| 50 | $-555.\overline{5}$ | $2,777.\overline{7}$ |
| 60 | 0 | 0 |

The magnitude of the flow rate is greatest at the beginning and gradually decreases to 0.

**50.** (a) $f'(8)$ is the rate of change of the quantity of coffee sold with respect to the price per pound when the price is \$8 per pound. The units for $f'(8)$ are pounds/(dollars/pound).

(b) $f'(8)$ is negative since the quantity of coffee sold will decrease as the price charged for it increases. People are generally less willing to buy a product when its price increases.

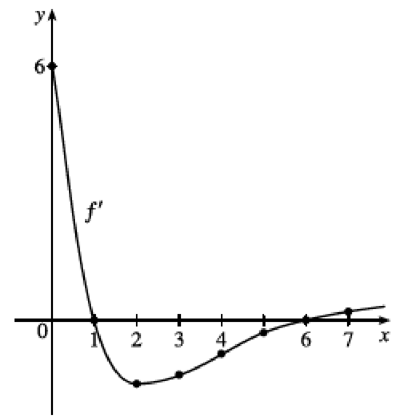
**2.** Your answers may vary depending on your estimates.

(a) *Note:* By estimating the slopes of tangent lines on the graph of $f$, it appears that $f'(0) \approx 6$.

(b) $f'(1) \approx 0$

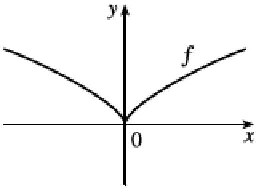
(c) $f'(2) \approx -1.5$ (d) $f'(3) \approx -1.3$ (e) $f'(4) \approx -0.8$

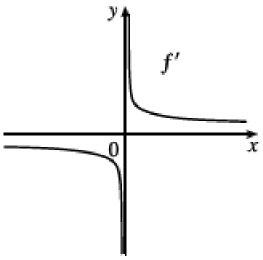
(f) $f'(5) \approx -0.3$ (g) $f'(6) \approx 0$ (h) $f'(7) \approx 0.2$



Hints for Exercises 4 –11: First plot $x$-intercepts on the graph of $f'$ for any horizontal tangents on the graph of $f$. Look for any corners on the graph of $f$—there will be a discontinuity on the graph of $f'$. On any interval where $f$ has a tangent with positive (or negative) slope, the graph of $f'$ will be positive (or negative). If the graph of the function is linear, the graph of $f'$ will be a horizontal line.

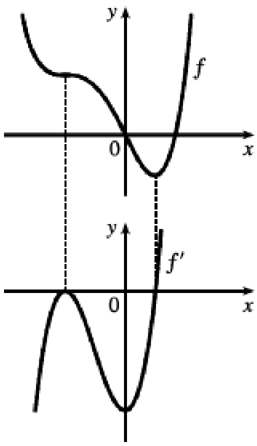
**8.**




Hints for Exercises 4 –11: First plot $x$-intercepts on the graph of $f'$ for any horizontal tangents on the graph of $f$. Look for any corners on the graph of $f$—there will be a discontinuity on the graph of $f'$. On any interval where $f$ has a tangent with positive (or negative) slope, the graph of $f'$ will be positive (or negative). If the graph of the function is linear, the graph of $f'$ will be a horizontal line.

**10.**




**24.** $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{\left[1.5(x+h)^2-(x+h)+3.7\right]-\left(1.5x^2-x+3.7\right)}{h}$

$= \lim\limits_{h\to 0} \dfrac{1.5x^2+3xh+1.5h^2-x-h+3.7-1.5x^2+x-3.7}{h} = \lim\limits_{h\to 0} \dfrac{3xh+1.5h^2-h}{h}$

$= \lim\limits_{h\to 0}(3x+1.5h-1) = 3x-1$

Domain of $f$ = domain of $f' = \mathbb{R}$.

**26.** $g'(t) = \lim_{h \to 0} \frac{g(t+h) - g(t)}{h} = \lim_{h \to 0} \frac{\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}}{h} = \lim_{h \to 0} \frac{\frac{\sqrt{t} - \sqrt{t+h}}{\sqrt{t+h}\,\sqrt{t}}}{h} = \lim_{h \to 0} \left( \frac{\sqrt{t} - \sqrt{t+h}}{h\,\sqrt{t+h}\,\sqrt{t}} \cdot \frac{\sqrt{t} + \sqrt{t+h}}{\sqrt{t} + \sqrt{t+h}} \right)$

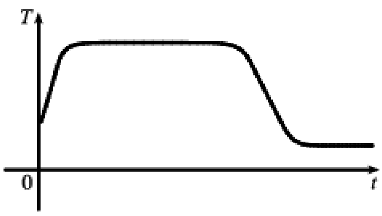
$= \lim_{h \to 0} \frac{t - (t+h)}{h\,\sqrt{t+h}\,\sqrt{t}\,(\sqrt{t} + \sqrt{t+h})} = \lim_{h \to 0} \frac{-h}{h\,\sqrt{t+h}\,\sqrt{t}\,(\sqrt{t} + \sqrt{t+h})} = \lim_{h \to 0} \frac{-1}{\sqrt{t+h}\,\sqrt{t}\,(\sqrt{t} + \sqrt{t+h})}$

$= \frac{-1}{\sqrt{t}\,\sqrt{t}\,(\sqrt{t} + \sqrt{t})} = \frac{-1}{t\,(2\,\sqrt{t})} = -\frac{1}{2t^{3/2}}$

Domain of $g$ = domain of $g' = (0, \infty)$.

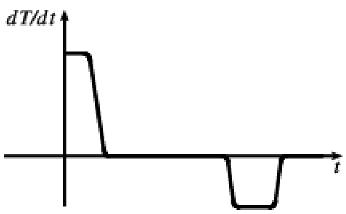
**44.** Where $d$ has horizontal tangents, only $c$ is 0, so $d' = c$. $c$ has negative tangents for $x < 0$ and $b$ is the only graph that is negative for $x < 0$, so $c' = b$. $b$ has positive tangents on $\mathbb{R}$ (except at $x = 0$), and the only graph that is positive on the same domain is $a$, so $b' = a$. We conclude that $d = f$, $c = f'$, $b = f''$, and $a = f'''$.

**58.** (a)

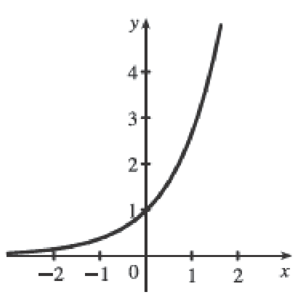



(b) The initial temperature of the water is close to room temperature because of the water that was in the pipes. When the water from the hot water tank starts coming out, $dT/dt$ is large and positive as $T$ increases to the temperature of the water in the tank. In the next phase, $dT/dt = 0$ as the water comes out at a constant, high temperature. After some time, $dT/dt$ becomes small and negative as the contents of the hot water tank are exhausted. Finally, when the hot water has run out, $dT/dt$ is once again 0 as the water maintains its (cold) temperature.

(c)




**2.** (a)




The function value at $x = 0$ is 1 and the slope at $x = 0$ is 1.

(b) $f(x) = e^x$ is an exponential function and $g(x) = x^e$ is a power function. $\frac{d}{dx}(e^x) = e^x$ and $\frac{d}{dx}(x^e) = ex^{e-1}$.

(c) $f(x) = e^x$ grows more rapidly than $g(x) = x^e$ when $x$ is large.

**6.** $F(x) = \frac{3}{4}x^8 \quad \Rightarrow \quad F'(x) = \frac{3}{4}(8x^7) = 6x^7$

**12.** $B(y) = cy^{-6} \quad \Rightarrow \quad B'(y) = c(-6y^{-7}) = -6cy^{-7}$

**16.** $h(t) = \sqrt[4]{t} - 4e^t = t^{1/4} - 4e^t \quad \Rightarrow \quad h'(t) = \frac{1}{4}t^{-3/4} - 4(e^t) = \frac{1}{4}t^{-3/4} - 4e^t$

**22.** $y = \dfrac{\sqrt{x} + x}{x^2} = \dfrac{\sqrt{x}}{x^2} + \dfrac{x}{x^2} = x^{1/2-2} + x^{1-2} = x^{-3/2} + x^{-1} \quad \Rightarrow \quad y' = -\frac{3}{2}x^{-5/2} + (-1x^{-2}) = -\frac{3}{2}x^{-5/2} - x^{-2}$

**26.** $k(r) = e^r + r^e \quad \Rightarrow \quad k'(r) = e^r + er^{e-1}$

**30.** $v = \left(\sqrt{x} + \dfrac{1}{\sqrt[3]{x}}\right)^2 = \left(\sqrt{x}\right)^2 + 2\sqrt{x} \cdot \dfrac{1}{\sqrt[3]{x}} + \left(\dfrac{1}{\sqrt[3]{x}}\right)^2 = x + 2x^{1/2-1/3} + 1/x^{2/3} = x + 2x^{1/6} + x^{-2/3} \quad \Rightarrow$

$v' = 1 + 2\left(\frac{1}{6}x^{-5/6}\right) - \frac{2}{3}x^{-5/3} = 1 + \frac{1}{3}x^{-5/6} - \frac{2}{3}x^{-5/3}$ or $1 + \dfrac{1}{3\sqrt[6]{x^5}} - \dfrac{2}{3\sqrt[3]{x^5}}$

**36.** $y = x^2 - x^4 \quad \Rightarrow \quad y' = 2x - 4x^3$. At $(1, 0)$, $y' = 2 - 4 = -2$ and an equation of the tangent line is $y - 0 = -2(x - 1)$ or $y = -2x + 2$. The slope of the normal line is $\frac{1}{2}$ (the negative reciprocal of $-2$) and an equation of the normal line is $y - 0 = \frac{1}{2}(x - 1)$ or $y = \frac{1}{2}x - \frac{1}{2}$.

**44.** $G(r) = \sqrt{r} + \sqrt[3]{r} \quad \Rightarrow \quad G'(r) = \frac{1}{2}r^{-1/2} + \frac{1}{3}r^{-2/3} \quad \Rightarrow \quad G''(r) = -\frac{1}{4}r^{-3/2} - \frac{2}{9}r^{-5/3}$

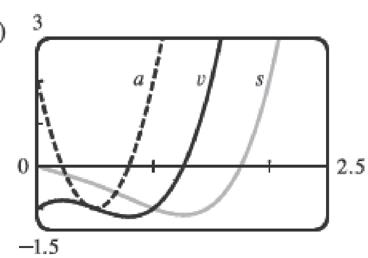
**48.** (a) $s = t^4 - 2t^3 + t^2 - t \quad \Rightarrow$

$v(t) = s'(t) = 4t^3 - 6t^2 + 2t - 1 \quad \Rightarrow$

$a(t) = v'(t) = 12t^2 - 12t + 2$

(b) $a(1) = 12(1)^2 - 12(1) + 2 = 2\,\text{m/s}^2$

(c)




**54.** $y = x\sqrt{x} = x^{3/2} \quad \Rightarrow \quad y' = \frac{3}{2}x^{1/2}$. The slope of the line $y = 1 + 3x$ is $3$, so the slope of any line parallel to it is also $3$. Thus, $y' = 3 \quad \Rightarrow \quad \frac{3}{2}x^{1/2} = 3 \quad \Rightarrow \quad \sqrt{x} = 2 \quad \Rightarrow \quad x = 4$, which is the $x$-coordinate of the point on the curve at which the slope is $3$. The $y$-coordinate is $y = 4\sqrt{4} = 8$, so an equation of the tangent line is $y - 8 = 3(x - 4)$ or $y = 3x - 4$.

**66.** $y = ax^2 + bx + c \quad \Rightarrow \quad y'(x) = 2ax + b$. The parabola has slope 4 at $x = 1$ and slope $-8$ at $x = -1$, so $y'(1) = 4 \quad \Rightarrow$ $2a + b = 4$ **(1)** and $y'(-1) = -8 \quad \Rightarrow \quad -2a + b = -8$ **(2)**. Adding **(1)** and **(2)** gives us $2b = -4 \quad \Leftrightarrow \quad b = -2$. From **(1)**, $2a - 2 = 4 \quad \Leftrightarrow \quad a = 3$. Thus, the equation of the parabola is $y = 3x^2 - 2x + c$. Since it passes through the point $(2, 15)$, we have $15 = 3(2)^2 - 2(2) + c \quad \Rightarrow \quad c = 7$, so the equation is $y = 3x^2 - 2x + 7$.