*we will go over the solution on Monday 8-11pm at West Wing Cluster.

Concepts Weird Fibonacci Problem

Recall Fibonacci Numbers:

$$F_n = \begin{cases} 0 & \text{if } n = 0; \\ 1 & \text{if } n = 1; \\ F_{n-1} + F_{n-2} & \text{if } n > 1. \end{cases}$$

n	0	1	2	3	4	5	6	7
F_n	**0**	**1**	**1**	**2**	**3**	**5**	**8**	**13**
Ratio with Previous Term	*N/A*	*N/A*	*1*	*2*	*1.5*	*1.667*	*1.6*	*1.625*

Suppose that you know that $F_{n+1} \geq (1.6)F_n$ for n bigger than or equal to 4.

Prove that, $F_{n+1} \leq 1.7(F_n)$ for n bigger than or equal to 3.

Let $P(n) =$ "$F_{n+1} \leq 1.7(F_n)$"

Base case $P(3)$ true b/c $F_4 = 3 \leq 1.7F_3 = 1.7(2) = 3.4$

IH Assume $P(k)$ true for some $k \geq 3$, $k \in \mathbb{N}$

Case $k = 3$: $P(4)$ true b/c $F_5 = 5 \leq 1.7F_4 = 1.7(3) = 5.1$

case $k \geq 4$, we also know $F_{k+1} \geq (1.6)F_k$

By IH, $F_{k+1} \leq 1.7F_k$

WTS $F_{k+2} \leq 1.7F_{k+1}$

NOTE $F_{k+1} = F_{k+2} - F_k$, so $F_{k+2} - F_k \leq 1.7F_k$

$\Leftrightarrow F_{k+2} \leq 2.7F_k$

Also note that $1.6F_k \leq F_{k+1}$

$\Leftrightarrow 2.7F_k \leq \frac{2.7}{1.6}F_{k+1} = 1.6875F_{k+1} \leq 1.7F_{k+1}$

So $F_{k+2} \leq 2.7F_k \leq 1.7F_{k+1}$

$\Leftrightarrow F_{k+2} \leq 1.7F_{k+1}$ ✓