*we will go over the solution on Monday 8-11pm at West Wing Cluster.

Concepts Weird Bijection Problem

Construct a **bijection**

$$f:\mathbb{N}\to\mathbb{N}\times\mathbb{N}$$

By using prime factorization on the input. Prove that it is a bijection.

Let $(2^a)(3^b)(5^c)(7^d)(11^e)\dots$ be a prime factorization of $X\in\mathbb{N}$.

Then $(3^b)(5^c)(7^d)(11^e)\dots=\frac{X}{2^a}$.

Define $f(x)=\left(a+1,\ \frac{\frac{X}{2^a}+1}{2}\right)$.

$$\boxed{\text{The idea: } a\in\mathbb{N}\cup\{0\},\text{ so } a+1\in\mathbb{N}.\quad \frac{X}{2^a}\in\{1,3,5,7,\dots\}\text{ so }\frac{\frac{X}{2^a}+1}{2}\in\mathbb{N}.}$$

INJ Let $X,Y\in\mathbb{N}$ be given where $X=2^a\dots$, $Y=2^{a'}\dots$ in prime factorization

Assume $f(x)=f(y)$.

This indicates that $a+1=a'+1\Rightarrow a=a'$

and $\frac{\frac{X}{2^a}+1}{2}=\frac{\frac{Y}{2^{a'}}+1}{2}\Rightarrow X=Y$ ✓

SURJ Let $(x,y)\in\mathbb{N}\times\mathbb{N}$

Then $f(2^{X-1}(2Y-1))=(X,Y)$ because $Y\in\mathbb{N}\Rightarrow 2Y-1\in$ odd, 1st coordinate must be $X-1+1=X$ and 2nd coordinate is $\frac{\frac{2^{X-1}(2Y-1)}{2^{X-1}}+1}{2}=Y$