21-378 **Mathematics of Fixed Income Markets** **D. Handron**

Midterm Exam Formula Sheet Fall, 2018

If the price P of a security can be described as a function of an interest rate factor y, $P = f(y)$, then

$$DV01 = -\frac{f'(y)}{10,000} \approx -\frac{\Delta P}{10,000\Delta y}, \quad D = -\frac{f'(y)}{P} \approx -\frac{\Delta P}{P\Delta y}, \quad C = \frac{f''(y)}{P}$$

If

$$f(y) = \frac{F}{(1+\frac{y}{2})^{2T}} + F\frac{q}{2}\sum_{i=1}^{2T}\frac{1}{(1+\frac{y}{2})^i}$$

then

$$f'(y) = -F\left[\frac{q}{y^2}\left(1 - \frac{1}{(1+\frac{y}{2})^{2T}}\right) + \left(1 - \frac{q}{y}\right)\frac{T}{(1+\frac{y}{2})^{2T+1}}\right]$$

The convexity of a coupon bond is given by

$$C = \frac{F}{P(1+\frac{y}{2})^2}\left[\frac{T(T+\frac{1}{2})}{(1+\frac{y}{2})^{2T}} + \frac{q}{2}\sum_{i=1}^{2T}\frac{i(i+1)}{4(1+\frac{y}{2})^i}\right]$$

or by

$$C = \frac{F}{P}\left[\frac{2q}{y^3}\left(1 - \frac{1}{(1+\frac{y}{2})^{2T}}\right) - \frac{2q}{y^2}\frac{T}{(1+\frac{y}{2})^{2T+1}} + \left(1 - \frac{q}{y}\right)\frac{T^2 + \frac{1}{2}T}{(1+\frac{y}{2})^{2T+2}}\right].$$

The convexity of a zero-coupon bond is given by

$$C = \frac{T(T+\frac{1}{2})}{(1+\frac{y}{2})^2}$$

Duration and Macaulay duration are related by

$$D_{Mac} = \left(1 + \frac{y}{2}\right)D.$$

The price of a perpetuity that pays A every 6 months and has yield to maturity y is given by

$$P = \frac{2A}{y}.$$