Week #9 Homework: Due on Wednesday, October 24.

1. In this exercise, we will use a continuous compounding convention for interest rates. Consider a coupon bond with face value $F > 0$, and maturity $T > 0$ that pays coupons continuously at the annual rate of $q > 0$ per unit time. The price of this bond at $t = 0$ is P. The yield to maturity of the bond is the unique $y > 0$ satisfying

$$P = Fe^{-yT} + Fq \int_0^T e^{-yt} dt.$$

   Let $DV01(F, q, y, T)$ denote the yield-based DV01 for this bond.

   (a) Show that for fixed $F, q, y > 0$ with $q > y$, $DV01(F, q, y, T)$ increases with increasing T.

   (b) Show that for fixed $F, q, y > 0$ with $q < y$, there exists a critical maturity T^{**} (depending on q and y) such that $DV01(F, q, y, T)$ increases with increasing T when $T < T^{**}$ and decreases with increasing T when $T > T^{**}$..

2. What, if anything, is wrong with the following argument concerning yields to maturity for coupon bonds:

   (i) For a fixed maturity, increasing the coupon rate must increase the price.

   (ii) Prices move inversely with yields, so (for a fixed maturity) bonds with higher prices must have lower yields.

   (iii) Combinint (i) ad (ii) above, we conclude that for fixed maturity, bonds with higher coupons should have lower yields.

3. In class, we looked at hedging a short position on a 30-year annuity that made payments of \$3250 two times per year. We constructed a hedging portfolio using 2-, 5-, 10-, and 30-year par-coupon bonds by considering key rate shifts. In class, we assumed that the yield curve was initially flat at 5%.

   For this problem, you should repeat this exercise, but assume that the initial par-yield curve is given by

$$y_{pc}(t) = .04\left(\frac{30 - t}{30}\right) + .06\left(\frac{t}{30}\right).$$

   Find the face values needed for the 2-, 5-, 10-, and 30-year par coupon bonds for the DV01's for each key rate to be zero. You may use the spreadsheet posted on the Canvas site to help you.

4. In this problem we will use key rates that are spot rates to hedge a security using zero coupon bonds. For simplicity, we will use two key rates: the 2-year and 10-year spot rates. Let y_2^* and y_{10}^* denote the current spot rates for maturities 2 and 10 years. Let y_2 and y_{10}

denote the spot rates for these maturities at some future time. We will model the spot rate curve at that time by

$$\hat{r}_{new}(t) = \hat{r}(t) + Y_2(t)(y_2 - y_2^*) + Y_{10}(t)(y_{10} - y_{10}^*)$$

where $\hat{r}(t)$ denotes the current spot-rate curve and the functions Y_2 and y_{10} are given by

$$Y_2(t) = \left\{ \begin{array}{cc} 1 & 0 < t \leq 2 \\ 1 - \frac{1}{8}(t-2) & 2 < t \leq 10 \\ 0 & 10 < t \leq 30 \end{array} \right.$$

$$Y_{10}(t) = \left\{ \begin{array}{cc} 0 & 0 < t \leq 2 \\ \frac{1}{8}(t-2) & 2 < t \leq 10 \\ 1 & 10 < t \leq 30 \end{array} \right.$$

Suppose that

$$\hat{r}(t) = .04\left(\frac{30-t}{30}\right) + .06\left(\frac{t}{30}\right)$$

and you have just sold \$1,000,000 face of a 10-year coupon bond with coupon rate $q = .05$. Determine the face values of the 2- and 10-year zeros needed to hedge your short position.

5. Repeat the problem above but this time assume that you have sold a 30-year annuity making \$5000 payments twice per year, and that

$$\hat{r}(t) = .06 - .06e^{-t/5}.$$