**Week #6 Homework**: Due on Wednesday, October 3.

1. Consider an annuity that pays a fixed amount $A$ every six months for $T$ years (where $T$ is a multiple of six months). Let $y$ be the yield to maturity for this annuity (under a semiannual compounding convention) so that

$$P = \sum_{i=1}^{2T} \frac{A}{(1 + \frac{y}{2})^i}.$$

   (a) Show that the duration and the Macauley duration for this annuity are given by

$$D = \frac{1}{y} - \left(\frac{1}{1 + \frac{y}{2}}\right) \left[\frac{T}{(1 + \frac{y}{2})^{2T} - 1}\right].$$

   and

$$D_{Mac} + \frac{1}{y} + \frac{1}{2} - \frac{T}{(1 + \frac{y}{y})^{2T} - 1}.$$

   (b) Find an expression for the yield based DV01 for this annuity.

   (c) Find an expression for the convexity of the annuity.

2. Let real numbers $y > 0$ and $T_1$, $T_2$, and $T_3$ be given with $0 < T_1 < T_2 < T_3$. You may assume that each $T_i$ is a multiple of six months. Assume tht the spot rate curve is flat at level $y$. Let three more real numbers $P$, $\delta$, and $\gamma$ be given, with $P > 0$.

   Show that there is a portfolio consisting of zero coupon bonds with maturities $T_1$, $T_2$, and $T_3$ such that the price of the portfolio is $P$ the Macauley duration of the portfolio is $\delta$ and the convexity of the portfolio is $\gamma$. (Some of the bond positions may be short, but the value of the portfolio as a whole should equal $P > 0$.)

   There is a result about invertibility of a special kind of matrix that is very helpful for this problem. I will post this result as a hint later.

   Note: The file `hw6spotRates.csv` gives the spot rates for maturities up to 10 years in two months, May and the following November, six months later. Use this data to answer the questions that follow.

3. Consider a coupon bond issued in May, with coupon rate $q = 3\%$ and maturity $T = 10$.

   (a) Determine the price of this bond in May, when it is issued. What is its yield to maturity at that time?

   (b) What is the yield based DV01 of this bond?

   (c) Find the duration and Macauley duration of the bond.

   (d) Find the convexity of the bond.

(e) What is the price of the bond in November? What is the change in price $\Delta P$ between May and November?

4. Let $T_d$ be the multiple of six months that is closest to the Macauley duration of the bond in Problem #3.

   (a) What is the yield to maturity $y_d$ of a zero coupon bond with maturity $T_d$?

   (b) What is the DV01 of this zero?

   (c) Suppose that it is May and you own the bond in Problem #3. You want to hedge you interest rate risk due to this bond. To do so, you plan to take a position in zero coupon bonds with maturity $T_d$ so that the DV01 of the combined portfolio will be 0. Should you buy or sell the zeros? How much face of the zeros should you hold?

   (d) What is the net change in value of the combined portfolio between May and November?

5. In this problem you will consider a different hedge from the one in the previous problem. Let $Z^2$, $Z^5$, and $Z^{10}$ be zero coupon bonds each with face value \$1 and with maturities of 2, 5, and 10 years respectively

   (a) Construct a portfolio from these three zeros that has the same price, Macauley duration, and convexity as the bond in Problem #3. For each bond, is your position long or short? How much face do you hold of each bond?

   (b) Suppose that it is May and you own the bond in Problem #3. You want to hedge you interest rate risk due to this bond. To do so, you plan to sell the portfolio of zeros you found in part (a). What is the net change in the value of your combined portfolio between May and November?

6. Which of the two hedges you constructed in this assignment did the best job of hedging your interest rate risk?