21-378 **Mathematics of Fixed Income Markets** **D. Handron**

Week #5 Homework: Due on Wednesday, September 26.

1. Use the data contained in the file `hw5spotRates.csv`, which lists the spot rates $\hat{r}(t)$ for $t \in \{.5, 1, 1.5, \ldots, 30\}$, to complete this problem

   (a) Using your favorite technology (e.g. Python, Excel, C++, R, Java, Pascal, BASIC, Fortran, COBOL...) produce a table that includes the maturities and spot rates given, along with the discount factors $d(t)$, the forward rates $f(t)$, and the par-coupon rates $y_{pc}(t)$ for each of the maturities listed.

   (b) Create a single plot that shows the spot rate curve, the forward rate curve, and the par-coupon yield curve for maturities up to 30 years.

   (c) For the maturities $t \in \{.5, 1, 1, 5, ...3\}$ compute the annuity yields $y_a(t)$ and create a single plot that shows the spot rate curve, the annuity yield curve, and the par-coupon yield curve for maturities up to 3 years.

   Note: it is not necessary to read the data in from the csv file. You can cut-and-paste to hard code it in.

2. A coupon bond has a flat price of 102.17 per hundred face. There are 182 days between the last coupon date and the next coupon date. 73 days have elapsed since the last coupon payment. The bond has a coupon rate of 5%. After the next coupon payment, the bond has 5 years until maturity (i.e., a person who buys the bond now will receive 11 coupon payments).

   (a) Determine the accrued interest for this bond as of today.

   (b) What is the full price of the bond today?

   (c) Find the yield to maturity of the bond.

3. (a) A certain T-Bill has 78 days until maturity. It's discount yield is 3.18%. What is the price of the bond? What is it's bond equivalent yield?

   (b) Another T-Bill has 221 days until maturity. The discount yield of this bond is 3.72%. What is the price of this bond? What is it's bond equivalent yield?

4. An *interest rate swap* is an agreement made at time 0 between two parties, who we will call Aidan and Brenda. It is characterized by a maturity T (assumed to be a multiple of 6 months), a notional principal amount $F > 0$, and a swap rate q. At each of the times $t = .5, 1, 1.5, \ldots, T$

   Aidan pays Brenda the variable amount $F\frac{r_{t-.5,t}}{2}$, and

   Brenda pays Aidan the constant amount $F\frac{q}{2}$.

where $r_{t-.5,t}$ is the spot rate that will prevail at time $t - .5$ for investments initiated at $t - .5$ and repaid at time t (using a semiannual compounding convention).

Consider a *perpetual* interest rate swap, which has no maturity (or has $T = \infty$). How could you replicate such a contract? What instruments would you use? What can you say about the swap rate q for such a contract?