21-378 Mathematics of Fixed Income Markets

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Week #3 Homework: Due on Wednesday, September 12.

1. (a) Three bonds, the $3\frac{1}{2}$ s of 11/15/2018, the 0s of 5/15/2019, and the $2\frac{1}{4}$ s of 11/15/2019, are trading on 5/15/2018. Their prices are given in the table below.

Bond	PRICE (as of $5/15/2018$)
$3\frac{1}{2}$ s of $11/15/2018$	100.71
$\bar{0}s \text{ of } 5/15/2019$	97.74
$2\frac{1}{4}$ s of $11/15/2019$	99.72

Determine the discount factors d(.5), d(1), and d(1.5) on that date (5/15/2018).

- (b) Suppose there existed a Treasury security with a 1% coupon maturing on November 15, 2019. Using the discount factors from part (a) what is the arbitrage free price of the 1s of 11/15/2019?
- (c) Suppose that the 1s of 11/15/2019 were trading at a price of 98.05 instead of the price you found in part (b). How could an arbitrageur profit from this price difference using the bonds priced in part (a). What would the profit be? (How should you measure the profit?)
- 2. (a) Given the two bonds priced in the table below, determine the arbitrage-free price of the third bond.

Bond	PRICE (as of $5/15/2010$)
0s of 5/15/2020	74.41
$2\frac{1}{2}$ s of 5/15/2020	?
$5\frac{1}{4}$ s of $5/15/2020$	115.33

- (b) Assuming that the $2\frac{1}{2}$ s of 5/15/2020 is the on-the-run 10-year note, why might the market price be different from the arbitrage-free price? Is the market price likely to be lower or higher than the arbitrage-free price?
- 3. (a) Given the discount factors d(.5) = .989711, d(1) = .977422, and d(1.5) = .964203, determine the spot rates $\{\hat{r}(.5), \hat{r}(1), \hat{r}(1.5)\}$ and the forward rates $\{f(.5), f(1), f(1.5)\}$.
 - (b) Are the forward rates above or below the spot rates in part (a)? Why is this the case?
- 4. Let $0 \le \eta \le T$, where η and T (measures in years) are both multiples of $\frac{1}{2}$. Find an expression for $r_{0,\eta,T}^{for}$ in terms of the forward rates $\{f(.5), f(1), f(1.5), \ldots, f(T)\}$.
- 5. Show that $r_{0,\eta,T}^{for} > 0$ if and only if $d(\eta) > d(T)$, i.e. that discount factors are a decreasing function of maturity if and only if forward rates are positive.