

Week #10 Homework: Due on Wednesday, October 31.

1. Assume the spot-rate curve is currently flat at 5%. A portfolio contains three zero coupon bonds, each with face value \$1,000,000. The maturities of the bonds are 2-, 7-, and 10-years.
 - (a) Find the DV01 and duration of the portfolio.
 - (b) Suppose that you would like to purchase a 5-year par-coupon bond with the same DV01 as the portfolio of zeros. What should the face value of the par-coupon bond be?
 - (c) Suppose the 2-year spot-rate increases by 45 basis points, the 7-year spot-rate increases by 11 basis points and the 10-year spot rate increases by 23 basis points. Compute the exact change in the price of the bond portfolio.
 - (d) What parallel shift in the yield curve would produce the same change in the value of the bond portfolio as in part (c)? [Use a first-order approximation to answer this question.]
2. Today the spot-rates for maturities 2-years and 10-years are $\hat{r}(2) = .035$ and $\hat{r}(10) = .05$. Some important economic news will be announced tomorrow, and you expect that after the announcement the 2-year rate will rise relative to the 10-year rate. You are uncertain, however, whether the rates will rise or fall.

Let $\hat{r}_{new}(2)$ and $\hat{r}_{new}(10)$ denote the spot rates after the announcement. Do not worry about the fact that a bond issued today with maturity T years will have a slightly shorter maturity tomorrow.

- (a) Suggest a trade involving \$1,000,000 face of a 2-year ZCB and some amount of face of a 10-year ZCB that will show a profit if $\hat{r}_{new}(10) - \hat{r}_{new}(2) < \hat{r}(10) - \hat{r}(2) = .015$. Specify the face value of the 10-year bond, and whether each bond is long or short. [Use a first-order approximation]
 - (b) What will the approximate value of your profit be if $\hat{r}_{new}(10) - \hat{r}_{new}(2) = .0139$, i.e. the 2-year rate rises 11 basis points relative to the 10-year rate. [Use a first-order approximation]
 - (c) What impact will the difference in the convexities of the 2-year and 10-year bonds have on the profit of your trade?
3. Imagine that you have been monitoring the spot rates for three relative maturities: the 2-year, 5-year, and 10-year rates. In the language of principal component analysis from class, $T_1 = 2$, $T_2 = 5$, and $T_3 = 10$. After observing the changes in rates for an appropriate amount of time, you compute a sample covariance matrix V (a 3×3 matrix). The eigenvalue of this matrix are $\lambda_1 > \lambda_2 > \lambda_3 > 0$. The corresponding eigenvectors are

$$\mathbf{e}^{(1)} = \frac{1}{3} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \quad \mathbf{e}^{(2)} = \frac{1}{3} \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{e}^{(3)} = \frac{1}{3} \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix},$$

Currently, the (semiannually compounded) spot rates are $\hat{r}(2) = .02$, $\hat{r}(5) = .075$, and $\hat{r}(10) = .03$.

A trader is short \$1,000,000 face of the 5-year zero, and wishes to hedge the short position by purchasing 2-year and 10-year zeros. Using the first two principal components, determine how much face of each bond the trader should buy.

4. The file `hw10yields.csv` contains three months worth of data for the following maturities: $T_1 = 2$, $T_2 = 3$, $T_3 = 5$, $T_4 = 7$, $T_5 = 10$, $T_6 = 20$, and $T_7 = 30$. Perform the following tasks with this data.
 - (a) Create a table with a column for each maturity. Each row of the table will contain the changes in yield from one trading day to the next. This table will have one fewer row than the table in `hw10yields.csv`.
 - (b) Compute the sample covariance matrix V for the rate changes.
 - (c) Compute the eigenvalues for the matrix V . Compute the eigenvectors corresponding to the three largest eigenvalues. Normalize each eigenvector so that it has unit length.
 - (d) Plot a curve for each of the three eigenvectors: for the eigenvector $\mathbf{e}^{(i)}$, plot the seven points $(t_j, \mathbf{e}_j^{(i)})$ for $j = 1, \dots, 7$. Connect these points with a straight line. You can include all three curves in a single plot, or you can make three separate plots.

You can use your favorite software package to complete this problem, and you may use different environments to complete different parts of the problem if you like.