21-378 **Mathematics of Fixed Income Markets** **D. Handron**

Final Exam Formula Sheet Fall 2018

Throughout the exam:

R_n is the spot rate for investing from time n to time $n+1$.
$B_{n,m}$ is the price at time n of a zero coupon bond with maturity m and face value \$1.
$F_{n,m}$ is the forward interest rate agreed to at time n for investments from time m to time $m+1$.
$SR_{n,m}$ is the swap rate agreed to at time n for an interest rate swap initiated at time n (the first payment is made at time $n+1$) with maturity m.
$\text{For}_{n,m}$ is the forward price agreed to at time n for the purchase of an asset at time m.
$\text{Fut}_{n,m}$ is the futures price agreed to at time n for the purchase of an asset at time m.
P is the initial principal of a T-year mortgage.
y is the mortgage rate under the monthly compounding convention.
$I(n)$ is the interest payment due at time n for a mortgage or a pool of mortgages.
$P(n)$ is the principal payment due at time n for a mortgage or a pool of mortgages.
$B(n)$ is the remaining principal balance after all payments at time n.

$$D_0 = 1, \quad D_n = \frac{1}{(1+R_0)(1+R_1)\dots(1+R_{n-1})}$$

$$V_n = \frac{1}{D_n}\widetilde{\mathbb{E}}_n[D_m V_m], \quad V_n = \frac{1}{1+R_n}\widetilde{\mathbb{E}}_n[V_{n+1}]$$

$$\frac{1}{D_n}\widetilde{\mathbb{E}}_n[D_m R_{m-1}] = B_{n,m-1} - B_{n,m}$$

$$SR_{n,m} = \frac{1 - B_{n,m}}{\sum_{i=n+1}^{m} B_{n,i}}$$

$$\text{Fut}_{n,m} = \widetilde{\mathbb{E}}[P_m]$$

$$\text{For}_{n,m} = \frac{\widetilde{\mathbb{E}}[D_m P_m]}{\widetilde{\mathbb{E}}[D_m]} = \text{Fut}_{n,m} + \frac{\text{Cov}(D_m, P_m)}{\widetilde{\mathbb{E}}[D_m]}$$

$$P = \sum_{i=1}^{12T} \frac{A}{(1+\frac{y}{12})^i} = A\lambda\left(\frac{1-\lambda^{12T}}{1-\lambda}\right), \quad \lambda = \frac{1}{1+\frac{y}{12}}$$

$$I(n+1) = B(n)\frac{y}{12}$$

$$P(n+1) = A - I(n+1)$$

$$B(n+1) = B(n) - P(n+1) - [\text{prepayments}]$$

$$CPR = 1 - (1 - SMM)^{12}, \quad SMM = 1 - (1 - CPR)^{\frac{1}{12}}$$