In the problems below: Today's date is $t = 0$. All coupon bonds and annuities make payments every 6 months, with the first payment to be made 6 months from today. All interest rates, coupon rates, and yields to maturity are expressed as annualized rates and the semiannual compounding convention is employed. All interest rates and yields should be assumed to be strictly positive. The payment amounts for annuities and the coupon rates for coupon bonds are strictly positive. We use $r_{t-.5,t}$ to denote the spot rate that will prevail at time $t - .5$ for loans initiated at time $t - .5$ and settled with a single payment at time t.

1. The table below gives prices (per \$100 face) of several bonds. Notice that two of the bonds pay coupons.

Maturity	**Coupon Rate**	**Price**
.5 years	0	97.947
1 year	.04	98.487
1.5 years	.06	101.80

   (a) Find the discount factors $d(.5)$, $d(1)$, and $d(1.5)$.

   (b) Find the spot rate $\hat{r}(1.5)$ and the forward rate $f(1)$.

   (c) Find the par-coupon yield $y_{pc}(1.5)$ for maturity 1.5.

2. A coupon bond with face value \$10,000, maturity 20 years, and annual coupon rate .08 is currently priced at \$11,245.36.

   (a) Write an equation for the yield to maturity y of the bond. (Do not solve the equation.)

   (b) Will the yield of the bond be above or below .08. Explain.

3. The annuity yield for maturity 10 years is $y_a(10) = .0468$. The yield to maturity of a coupon bond having coupon rate .06 and maturity 10 years is $y = .05$. Find the par-coupon yield $y_{pc}(10)$ for maturity 10 years or explain what it is not possible to determine $y_{pc}(10)$ from the given information.

4. Bond #1 is a coupon bond with coupon rate $q_1 = .04$ and maturity 10 years. Bond #2 is a coupon bond with coupon rate $q_2 = .06$ and maturity 10 years. The yield to maturity of Bond #1 is $y_1 = .05$ and the yield to maturity of Bond #2 is $y_2 = .054$. Find the par-coupon yield $y_{pc}(10)$ for maturity 10 years.

5. Assume that the spot-rate curve is flat at 4%. (We are not assuming that it will remain flat.) Find the time-0 price of a bond with face value \$100,000 and maturity 10 years that pays variable coupons $50,000(1.25)r_{t-.5,t}$ at each of the times $t = .5, 1, 1.5, 2, \cdots, 10$ plus a payment of 100,000 at time 10. (This bond pays coupons that are 25% higher than a standard floating rate bond.)

6. Suppose that the spot-rate curve is upward sloping. In particular, assume that $\hat{r}(t+.5) > \hat{r}(t)$ for $t = .5, 1, 1.5, \cdots, 10$. Consider the following securities:

   - Security #1 is a zero coupon bond with maturity 10 years.
   - Security #2 is an annuity with maturity 5 years.
   - Security #3 is an annuity with maturity 10 years.
   - Security #4 is a coupon bond with maturity 10 years.

   If possible, order these securities by yield-to-maturity from lowest to highest. Give a brief explanation of your ordering. (Equations not required.) If it is not possible to order the yields based on the information, give a brief explanation of why not.

7. Assume that the spot-rate curve is flat at 4%. Portfolio A holds a par coupon bond and a zero-coupon bond. The par-coupon bond has maturity $T_1 = 6$ and face value $F_1 = \$10,000,000$. The zero-coupon bond has maturity $T_2 = 20$ and face value $F_2 = \$18,000,000$.

   (a) Find the Macaulay duration and DV01 for portfolio A.

   (b) If the spot-rate curve undergoes a parallel shift of 14 basis points upward, by approximately how much will the price of portfolio A change? Be sure to specify both the magnitude and direction of the price change. (Use a first-order approximation.)

   (c) Portfolio B is to consist of a single par-coupon bond with maturity $T_3 = 10$ and face value F_3. Determine F_3 so that portfolios A and B have the same DV01.

8. Assume that the spot-rate curve is flat at 4%. Portfolio A holds three par coupon bonds: the first bond has maturity $T_1 = 10$ and face value $F_1 = \$15,000,000$, the second bond has maturity $T_2 = 20$ and face value $F_2 = \$10,000,000$, and the third bond has maturity $T_3 = 30$ and face value $F_3 = \$25,000,000$.

   (a) Find the Macaulay duration and $DV01$ for portfolio A.

   (b) If the spot-rate curve undergoes a parallel shift downward of 21 basis points, by approximately how much will the price of portfolio A change? Be sure to specify both the magnitude and direction of the price change. (Use a first-order approximation.)

   (c) Portfolio B is to consist of a two zero coupon bonds having the same face value F. One of these bonds has maturity $T_4 = 12$ years and the other has maturity $T_5 = 18$ years. The face value F is to be chosen so that portfolios A and B have the same DV01. Determine F.

9. Suppose that the spot rate curve is flat at some unspecified level $y > 0$. Consider the following three securities:

- Security #1 is an annuity with maturity 7 years that pays $\$10,000\frac{y}{2}$ at each of the times $.5, 1, 1.5, \cdots, 6.5, 7$
- Security #2 is a par-coupon bond with face value \$10,000 and maturity 7 years.
- Security #3 is a coupon bond with face value \$10,000 and maturity 7 years and time-0 price \$10,650.

(a) Order the securities by (Macaulay) duration from largest to smallest if possible, or explain why such an ordering is not possible based on the information given.

(b) Order the securities by convexity from largest to smallest if possible, or explain why such an ordering is not possible based on the information given.

(c) Order the securities by DV01 from largest to smallest if possible, or explain why such an ordering is not possible based on the information given.

10. Suppose that the spot rate curve is flat at some unspecified level $y > 0$. Consider the following three securities:

- Security #1 is a zero-coupon bond with face value 100,000 and maturity 20 years.
- Security #2 is a par-coupon bond with face value 100,000 and maturity 20 years.
- Security #3 is a zero coupon bond with face value 100,000 and maturity 30 years.

(a) Order the securities by (Macaulay) duration from largest to smallest if possible, or explain why such an ordering is not possible based on the information given.

(b) Order the securities by convexity from largest to smallest if possible, or explain why such an ordering is not possible based on the information given.

(c) Order the securities by DV01 from largest to smallest if possible, or explain why such an ordering is not possible based on the information given.

11. Consider a (nonstandard) 10-year interest-rate swap such at each of the times $t = .5, 1, 1.5, \cdots, 4.5, 5$

A pays B the amount $\dfrac{Fr_{t-.5,t}}{2}$ and

B pays A the amount $\dfrac{Fq}{2}$,

and at each of the times $t = 5.5, 6, 6.5, \cdots, 9.5, 10$

A pays B the amount Fq

B pays A the amount $Fr_{t-.5,t}$.

Notice that half way through the swap, the notional principal doubles and the roles of receiving floating and fixed are interchanged. The same swap rate q is used throughout the entire swap, and q is chosen so that neither party pays anything to enter into the agreement. (No notional principal is ever paid.) Find a formula for q in terms of the discount factors $d(.5), d(1), d(1.5), \cdots, d(10)$. Be sure to explain your reasoning fully and carefully.

12. (Riding the Yield Curve) Assume that the spot rate curve is upward sloping, In particular, assume that $\hat{r}(t+.5) > \hat{r}(t)$ for $t = .5, 1, 1.5, \cdots, 5$. Two investors A and B each invest the same amount of capital at time 0. Investor A buys a par-coupon bond with maturity 2 years, and investor B buys a par-coupon bond with maturity 5 years. Assume that both investors use the coupons received at times .5, 1, and 1.5 to purchase ZCBs maturing at time 2. If the term structure remains unchanged for two years, which investor will be better off in two years and why? In particular, you should assume that $r_{2,2+T} = \hat{r}(T)$ for $T = .5, 1, 1.5 \cdots, 5$. Here $r_{2,2+T}$ is the spot rate that will prevail at time 2 for borrowing or investing between time 2 and time $2+T$.