21-370 DISCRETE TIME FINANCE WRITTEN HOMEWORK — WEEK #4

2016 FALL

- (1) Exercise 1.8.
- (2) Exercise 2.1. For 2.1(ii) write a proof by induction.
- (3) Exercise 2.2.
- (4) In the framework of an N-period binomial model with the parameters u, d, and r, consider the down-and-in call option, which becomes the standard call option with strike price K and maturity N as soon as the stock's price, S_n goes below the lower barrier L. If S_n stays above L for all times n, then the option expires worthless.

Denote by $f_n(x)$ the option's price at time n if $S_n = x$ and the lower barrier has been crossed before n.

Denote by $g_n(x)$ the option's price at time *n* if $S_n = x$ and the lower barrier has been *not* crossed before *n*.

- (a) Find an expressions for f_N(x) in terms of x, K, and L. Find an expressions for g_N(x) in terms of x, K, L, and f_N(x). You may express g_N using characteristic functions (e.g. 1_{s∈A}(s)) or by cases.
- (b) Write the backward induction formulas needed to compute $f_n(x)$ and $g_n(x)$ when $n \in \{0, 1, ..., N-1\}$. Again, the formula for g_n may be refer to the function f_n .
- (c) Consider the model with N = 3, u = 2, $d = \frac{1}{2}$, $r = \frac{1}{4}$, and $S_0 = 4$.

For the down-and-in call with strike price K = 5 and lower barrier L = 3, find $V_0 = g_0(S_0)$, the arbitrage free price, and Δ_0 , the number of shares held in the replicating portfolio, at time 0.