21-270 **Introduction to Mathematical Finance**

Week #6 Homework: Due Wednesday, February 19 **Spring 2020**

1. Suppose that there is an ideal bank, but the spot rates are given as nominal rates $r[4]_*\left(\frac{i}{4}\right)$, $i \in \{1, 2, 3, \ldots\}$, corresponding to quarterly compounding, i.e. the discount factors are given by
$$D(T) = \frac{1}{\left(1 + \frac{r[4]_*(T)}{4}\right)^{4T}}$$
when T is an integer multiple of $\frac{1}{4}$.

   At $t = 0$, Aidan agrees to borrow \$100,000 at $t = 1$ and repay the loan with a single lump-sum payment of \$112,000 at $t = 3$. Given that $r[4]_*(1) = .06$ and there is no arbitrage, find $r[4]_*(3)$.

2. Annuities that will pay \$500 twice per year for the next 5 years (i.e., 10 payments in all) are being issued today at the arbitrage-free price of \$3850 per annuity. Coupon bonds having maturity 5 years and face value \$1,000 are being issued today at the arbitrage-free price \$1,050. These bonds pay coupons twice per year at the nominal coupon rate $q[2] = 10\%$. (The coupon payments are made on the same days as the annuity payments.). Find the effective 5-year sopt rate $R_*(5)$.

3. Suppose that there is an ideal money market with constant effective rate R. A customer calls the bank and asks: "If I deposit \$1,000 today, deposit an additional \$2,000 6 months from today, and make no other deposits or withdrawals, what will my account balance be two years from today?" The bank answers \$3215.65. Determine R.

4. Coupon bonds with maturity $T = 10$ years and face value \$5,000 that pay coupons twice per year at the nominal rate $q[2] = .06$ are currently trading at \$4851.20 per bond. Coupon bonds with maturity $T = 10$ and face value \$10,000 that pay coupons twice per year at the nominal rate $q[2] = .08$ are trading at \$11,261.32 per bond. Assuming that there is no arbitrage, determine the prices of each of the following fixed-income securities:

   (a) A zero-coupon bond with face value \$20,000 and maturity 10 years.

   (b) An annuity that has maturity 10 years and makes payments of \$500 twice per year.

   (c) A coupon bond with maturity 10 years and face value \$7,500 that pays coupons twice per year at the nominal rate $q[2] = .06$.

5. Find the effective yield to maturity, R_I, and the nominal yield to maturity, $r_I[2]$, for the three securities you priced in Problem #4. You may wish to use a numerical method to find the roots of the polynomial used to compute the yield for securities (b) and (c).