Week #4 Written Assignment: Due on Friday, September 20.

1. Consider the differential equation
$$y' = t^2 - y^2$$
   (a) Sketch the direction field for this equation. Show the isoclines for $m = 0, 1, -1$ and whatever other information is needed to give a good idea of how solutions behave.
   (b) I claim that the solution satisfying the initial condition $y(1) = 1/2$ also satisfies $y(t) > 0$ for *every* $t > 1$. Is this assertion correct? Justify your answer.
   (c) I claim that the solution satisfying $y(1) = 1/2$ also satisfies $y(t) > t$ for *some* $t > 1$. Is this assertion correct? Justify your answer.

2. A differential equation of the form
$$\frac{dy}{dx} + P(x)y = Q(x)y^n, \quad n \in \mathbb{R}$$
is called a *Bernoulli Equation*. Note that for $n \in \{0, 1\}$ the Bernoulli equation is linear.
   (a) Assume that $n \neq 0$ and $n \neq 1$. Show that the substitution $u = y^{1-n}$ results in a linear differential equation for u. [Hint: divide both sides of the Bernoulli equation by y^n.]
   (b) Solve the Bernoulli equation $\frac{dy}{dx} + 8y = e^{2x}y^5$.

3. The logistic equation
$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$$
is useful for modeling populations with limited resources. In some situations it is observed that populations below some threshold value will fail t thrive and will eventually become extinct. The logistic model can be modified to reflect this observation by adding an additional factor:
$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)\left(1 - \frac{m}{P}\right)$$
   (a) Assuming $0 < m < M$ sketch a graph of the function $f(x) = kx\left(1 - \frac{x}{M}\right)\left(1 - \frac{m}{x}\right)$. Your sketch should indicate where the function is positive, where it is negative, and where it is zero.
   (b) Draw the phase line for the modified logistic equation, $\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)\left(1 - \frac{m}{P}\right)$. Determine the stability of each equilibrium point.
   (c) Using the phase line as a guide, sketch a representative sample of solutions in the tP-plane. What does this model prediction about populations with $0 < P < m$? With $m < P < M$? With $M < P$?