Exam #2 Review, Fall 2018

1. Let
$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$
Find all the eigenvalues of the matrix A. For each eigenvalue, express the eigenspace as a span of vectors.

2. Let
$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & -2 \\ -2 & -4 & 4 \\ 1 & 2 & -2 \end{bmatrix}$$
   (a) Compute the product AB. Your response should make clear how you arrived at your result.
   (b) Is there an $n \times n$ matrix M such that $AM = B$? Either find such a matrix, or explain how you know there is not.

3. (a) Compute the determinant of the matrix
$$\begin{bmatrix} 3 & 6 & 9 \\ 2 & 4 & 9 \\ 1 & 4 & 2 \end{bmatrix}$$
by performing row operations to reduce it to a triangular form.
   (b) Determine conditions on the values a, b, and c that ensure the matrix
$$\begin{bmatrix} a & a & a \\ a & b & b \\ a & b & c \end{bmatrix}$$
is invertible.

4. Consider the matrix
$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 2 & 3 & a \end{bmatrix}$$
where $a \in \mathbb{R}$.
   (a) For what values of a will this matrix be invertible?
   (b) Suppose that $a = 1$. What is the inverse of the matrix A in this case?

5. Consider the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & -1 \\ 0 & 3 & 1 \end{bmatrix}$$

where $a \in \mathbb{R}$.

(a) $\mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector of A. What is it's corresponding eigenvalue?

(b) $\lambda = 2$ is an eigenvalue of A. Express the eigenspace E_2 as a span of vectors.

6. Consider the matrix

$$A = \begin{bmatrix} a & a & a \\ a & a^2 & a^2 \\ a & a^2 & a^3 \end{bmatrix}.$$

(a) Compute $\det(A)$ by expanding across the second row, using the Laplace expansion method. Simplify your answer by collecting terms and/or factoring.

(b) For what values of a does this matrix fail to be invertible?

7. (a) For what values of $a \in \mathbb{R}$ will the matrix

$$\begin{bmatrix} -3 & 0 & -2 \\ a & 2 & 0 \\ a & a & -1 \end{bmatrix}$$

fail to be invertible?

(b) The matrix

$$A = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$$

is invertible. Find it's inverse, A^{-1}.

8. Consider the matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

(a) Find all the eigenvalues of the matrix A.

(b) Describe each eigenspace of A as a span of vectors.

9. Let A and B be $n \times n$ matrices, and P an invertible $n \times n$ matrix such that

$$A = PBP^{-1}.$$

(a) Suppose that $\mathbf{v}$ is an eigenvector of A with corresponding eigenvalue λ, i.e. $A\mathbf{v} = \lambda\mathbf{v}$, with $\mathbf{v} \neq \mathbf{0}$. Show that λ is also an eigenvalue of B. Determine an eigenvector of B that corresponds to the eigenvalue λ.

(b) If

$$P = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 5 \end{bmatrix}, \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

and $\mathbf{v}$ is an eigenvector of A, find an eigenvector of the matrix B.