Final Exam — Reference Table

Definition 1 *Let V be a set with two operations, addition and scalar multiplication. If $\mathbf{u}, \mathbf{v} \in V$, we denote their sum as $\mathbf{u} + \mathbf{v}$. If $\mathbf{u} \in V$ and c is a scalar, we denote their scalar product as $c\mathbf{u}$.*

We say that V is a vector space *if the following axioms hold for all $\mathbf{u}, \mathbf{v}, \mathbf{w} \in V$ and all scalars c, d:*
[1.] $\mathbf{u} + \mathbf{v} \in V$, [2.] $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$, [3.] $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$, [4.] There exists a vector $\mathbf{0} \in V$ such that $\mathbf{u} + \mathbf{0} = \mathbf{u}$ for all vectors $\mathbf{u} \in V$, [5.] For each vector $\mathbf{u} \in V$ there is a vector $(-\mathbf{u}) \in V$ such that $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$, [6.] $c\mathbf{u} \in V$, [7.] $c(\mathbf{u}+\mathbf{v}) = c\mathbf{u} + c\mathbf{v}$, [8.] $(c+d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$, [9.] $c(d\mathbf{u}) = (cd)\mathbf{u}$, [10.] $1\mathbf{u} = \mathbf{u}$

Theorem 1 (The Fundamental Theorem of Invertible Matrices: Version III) *Let A be an $n \times n$ matrix and $T : V \to W$ a linear transformation whose matrix $[T]_{\mathcal{C} \leftarrow \mathcal{B}}$ with respect to the bases $\mathcal{B}$ of V and $\mathcal{C}$ of W is A. The following are equivalent*

[1.] A is invertible. [2.] $A\mathbf{x} = \mathbf{b}$ has a unique solution $\forall\, \mathbf{b} \in \mathbb{R}^n$. [3.] $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, $\mathbf{x} = \mathbf{0}$. [4.] The reduced row echelon form of A is I_n. [5.] A is a product of elementary matrices. [6.] rank$(A) = n$. [7.] nullity$(A) = 0$. [8.] The column vectors of A are linearly independent. [9.] The column vectors of A span $\mathbb{R}^n$. [10.] The column vectors of A form a basis for $\mathbb{R}^n$. [11.] The row vectors of A are linearly independent. [12.] The row vectors of A span $\mathbb{R}^n$. [13.] The row vectors of A form a basis for $\mathbb{R}^n$. [14.] $\det(A) \neq 0$. [15.] 0 is not an eigenvalue of A. [16.] T is invertible. [17.] T is one-to-one. [18.] T is onto. [19.] ker$(T) = \{\mathbf{0}\}$. [20.] range$(T) = W$. [21.] 0 is not a singular value of A.