Homework #10: Due on Friday, November 2.

1. (a) Let $\mathbf{v}$ and $\mathbf{w}$ be vectors in $\mathbb{R}^n$. Explain why $\mathbf{v} - \mathbf{w}$ can be thought of as an arrow pointing from $\mathbf{w}$ to $\mathbf{v}$. [Hint: think about the "tip-to-tail" rule for vector addition.]

   (b) Explain why $\{t\mathbf{v} + (1-t)\mathbf{w} : t \in [0, 1]\}$ is the set of vectors on the line segment with endpoints $\mathbf{v}$ and $\mathbf{w}$.

   (c) Let S be a subspace of $\mathbb{R}^n$, and suppose that $\mathbf{v}, \mathbf{w} \in S$. Show that S contains the line segment with endpoints $\mathbf{v}$ and $\mathbf{w}$.

2. [**Poole, Section 6.1, Problem #38**.] Let $\mathscr{F}$ denote the set of functions mapping $\mathbb{R}$ to $\mathbb{R}$. For $f, g \in \mathscr{F}$ and $c \in \mathbb{R}$ define $f \oplus g : \mathbb{R} \to \mathbb{R}$ and $c \odot f : \mathbb{R} \to \mathbb{R}$ by

$$(f \oplus g)(x) = f(x) + g(x)$$

   and

$$(c \odot f)(x) = c \cdot f(x)$$

   $\mathscr{F}$ is a vector space under these operations.

   Let $W = \{f \in \mathscr{F} : f(-x) = f(x)\}$. Determine whether W is a subspace of $\mathscr{F}$.

3. [**Poole, Section 6.1, Problem #48**.] Let V be a vector space with subspaces U and W. Define the *sum of* U *and* W to be

$$U + W = \{\mathbf{u} + \mathbf{w} : \mathbf{u} \in U, \mathbf{w} \in W\}$$

   (a) If $V = \mathbb{R}^3$, U is the x-axis, and W is the y-axis, what is $U + W$?

   (b) Show that for any subspaces U and W of a vector space V, $U + W$ is a subspace of V.

   (c)

4. (a) Let V be a vector space and $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\} \subseteq V$ a linearly independent set of vectors. Show that if $\mathbf{w} \in V$ and $\mathbf{w} \notin \operatorname{span}(\mathbf{v}_1, \ldots, \mathbf{v}_k)$ then $\{\mathbf{v}_1, \ldots, \mathbf{v}_k, \mathbf{w}\}$ is a linearly independent set.

   (b) Let W be a vector space and $\mathcal{S} = \{\mathbf{w}_1, \ldots, \mathbf{w}_k\}$ a set of vectors in W. Show there is a linearly independent subset of $\mathcal{S}$ that has the same span as $\mathcal{S}$.