

1. Suppose we have a necklace of n beads. Each bead is labeled with an integer and the sum of all these labels is $n - 1$. Prove that we can cut the necklace to form a string whose consecutive labels x_1, x_2, \dots, x_n satisfy

$$\sum_{i=1}^k x_i \leq k - 1 \quad \text{for } k = 1, 2, \dots, n.$$

2. Find the least number A such that for any two squares of combined area 1, a rectangle of area A exists such that the two squares can be packed in the rectangle (without interior overlap). You may assume that the sides of the squares are parallel to the sides of the rectangle.
3. Let $c > 0$ be a constant. Give a complete description, with proof, of the set of all continuous functions $f : R \rightarrow R$ such that $f(x) = f(x^2 + c)$ for all $x \in R$. Note that R denotes the set of real numbers.