

1. Right triangle  $ABC$  has right angle at  $C$  and  $\angle BAC = \theta$ ; the point  $D$  is chosen on  $AB$  so that  $|AC| = |AD| = 1$ ; the point  $E$  is chosen on  $BC$  so that  $\angle CDE = \theta$ . The perpendicular to  $BC$  at  $E$  meets  $AB$  at  $F$ . Evaluate  $\lim_{\theta \rightarrow 0} |EF|$ .
2. Let  $S$  be a set of real numbers which is closed under multiplication (that is, if  $a$  and  $b$  are in  $S$ , then so is  $ab$ ). Let  $T$  and  $U$  be disjoint subsets of  $S$  whose union is  $S$ . Given that the product of any *three* (not necessarily distinct) elements of  $T$  is in  $T$  and that the product of any three elements of  $U$  is in  $U$ , show that at least one of the two subsets  $T, U$  is closed under multiplication.
3. For a partition  $\pi$  of  $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ , let  $\pi(x)$  be the number of elements in the part containing  $x$ . Prove that for any two partitions  $\pi$  and  $\pi'$ , there are two distinct numbers  $x$  and  $y$  in  $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$  such that  $\pi(x) = \pi(y)$  and  $\pi'(x) = \pi'(y)$ . [A *partition* of a set  $S$  is a collection of disjoint subsets (parts) whose union is  $S$ .]