

1. Let f be a real function with a continuous third derivative such that $f(x), f'(x), f''(x), f'''(x)$ are positive for all x . Suppose that $f'''(x) \leq f(x)$ for all x . Show that $f'(x) < 2f(x)$ for all x .
2. Let \mathcal{F} be a finite collection of open discs in \mathbb{R}^2 whose union contains a set $E \subseteq \mathbb{R}^2$. Show that there is a pairwise disjoint subcollection D_1, \dots, D_n in \mathcal{F} such that

$$E \subseteq \cup_{j=1}^n 3D_j.$$

Here, if D is the disc of radius r and center P , then $3D$ is the disc of radius $3r$ and center P .