21-124 MODELING WITH DIFFERENTIAL EQUATIONS

LECTURE 9: CHAOTIC SYSTEMS
EXPERIMENTS AND DISCUSSION

1. The Lorenz System

The Lorenz System is described by the differential Equations

$$\begin{aligned} \tfrac{dx_1}{dt} &= -Ax_1 + Ax_2 \\ \tfrac{dx_2}{dt} &= Rx_1 - x_2 - x_1x_3 \\ \tfrac{dx_3}{dt} &= -Bx_3 + x_1x_2. \end{aligned}$$

We will consider the case where $A = 10$, $B = 8/3$ and R takes values in the set $\{.5, 23, 28\}$. Use the M-file

```
function y=lor(t,u)
global R;
y=zeros(3,1);
y(1)=10*(-u(1)+u(2));
y(2)=R*u(1)-u(2)-u(1).*u(3);
y(3)=-(8/3)*u(3)+u(1).*u(2);
```

With MATLAB's numerical solvers to investigate the behavior of this system. R has been delcared as a global variable in this M-file. Enter the same command in your MATLAB window. Then set `R=.5`.

1. Calculate for $0 \le t \le 30$ a solution `y` satisfying the initial conditions $[1, 1, 1]$. Use `rk4` with a step size of $(.1)$.
2. Calculate for $0 \le t \le 30$ a solution `z` satisfying the initial conditions $[1, 1, 1.01]$. Use `rk4` with a step size of $(.1)$.
3. Graph, on the same set of axes, the functions
$$\begin{aligned} &y_1(t) - z_1(t) \\ &y_2(t) - z_2(t) \\ &y_3(t) - z_3(t). \end{aligned}$$
This can be done with the command `plot(t,y-z)`. What is this showing you about the solutions?
4. How large does the difference get? Does the difference increease slowly or quickly?

Repeat these steps for the valuse $R = 23$ and $R = 28$.

Chaos is characterized by a tendancy for slolutions with similar initial conditions to follow each other closely for a while, and then quickly diverge wildly. For which values of the parameter R would you say the system exhibits *chaos*?