21-124 MODELING WITH DIFFERENTIAL EQUATIONS

LECTURE 8: AUTONOMOUS SYSTEMS AND THE PREDATOR-PREY EQUATIONS
EXPERIMENTS AND DISCUSSION

1. Arms Races

The first few questions will consider a hypothetical arms race between Florin and Guilder. The basic model assumes that each country wants to spend twice as much as the other on weapons, but that spending is limited by the economic resources available to each country. The model is

$$\left.\begin{aligned} \frac{df}{dt} &= \left(g - \frac{f}{2}\right) - f^2 \\ \frac{dg}{dt} &= \left(f - \frac{g}{2}\right) - g^2 \end{aligned}\right\}$$

where $f(t)$ is the annual military budget of Florin and $g(t)$ is the annual military budget of Guilder (in billions of US$).

In each problem below, the assumptions of the model are modified somewhat. For each of these modifications, you should (a) make the corresponding modification to the system above; (b) have `pplane` create the vector field for the modified model and plot enough solutions to "get a feel" for what is going on; (c) find the equilibrium points of the system (algebraically if it is not too difficult, or approximately, by inspection of the phase plane otherwise); (d) describe what will happen if the arms race continues indefinitely (this may depend on the initial conditions chosen, indicate so if this is the case).

Treat each case as a separate modification of the *original* model. Don't treat them as cumulative modifications.

1. Suppose that both countries decide they need only match the spending of the other nation, not double it. (All other assumptions remain unchanged.)

2. Suppose Florin decides it only needs to match Guilder's spending, but Guilder is still determined to double Florin's spending.

3. Suppose the economic pressure against stockpiling arms is unchanged in Guilder, but doubles in Florin.

4. Suppose gold is discovered in Guilder, removing the economic pressures which limit military spending.

5. Suppose both countries become so wealthy that neither has any economic pressure to limit spending.