# 21-124 MODELING WITH DIFFERENTIAL EQUATIONS

HOMEWORK 5: DUE ON FRIDAY, MARCH 15 BY 5:00PM

In this set of problems, we will examine the system

$$\frac{dx}{dt} = -y - z$$
$$\frac{dy}{dt} = x + Ay$$
$$\frac{dz}{dt} = B + z(x - C).$$

This is the Rossler system and for some values of the parameters it displays chaotic behavior similar to that of the Lorenz system. We will investigate the behavior when $B = 2$, $C = 3$ and $A$ varies between 0.3 and 0.4.

You will need to download the M-file rossler.m from the course website. This m-file sets $B = 2$ and $C = 3$, but makes use of a *global* variable, $A$. When you sit down to work on these problems, run MATLAB and begin by entering `global A` on the command line. Then you can define and redefine the value of A as always. When you (or one of the solvers) calls the function rossler, it will use the value of A defined most recently.

You may want to work through all three problems before you begin your final write-up.

1. **(Period Doubling)**
   (a) Use `ode45` to compute a numerical solution to the system with $A = 0.3$. Use $[1.2; 1.2; 1.2]$ as the initial condition, and compute the solution from $t = 0$ to $t = 100$. That is, enter the command `[t,y]=ode45('rossler',[0,100],[1.2,1.2,1.2])`. Use the command `plot(t,y)` to graph the solutions. Do they look periodic (or do they settle down to a periodic function)? You may want to look at each solution individually with the command `plot(t,x(:,i))` where $i = \{1, 2, 3\}$.
   (b) Use the command `plot3(y(:,1),y(:,2),y(:,3))` to produce a phase space graph of the solution you computed. Now compute a new solution, using the last line of the matrix `y` as the initial conditions. Look at the three dimensional plot of this solution. It should display a single loop. This solution curve represents a periodic solution.
   (c) Change the value of $A$ to 0.35 and compute another solution `[s,z]`. As above, use the last line of the matrix `z` as the initial condition another solution. Use `plot3` to display this solution curve. This curve should be a "double loop". As $A$ changes from .3 to .35, the period has doubled.
   (d) Change the value of $A$ to 0.375 and repeat the steps in part 1c. The period has doubled again. Now the solution travels four times around before repeating.

2. **(Chaos)** Now change the value of $A$ to 0.398. Compute a solution `[t,y]` with initial condition $[1.2, 1.2, 1.2]$ and a solution `[t,z]` with initial condition $[1.2, 1.2, 1.21]$. Use the command `plot(t,y-z)` to look at the difference between these solutions. Would you say this system displays chaotic behavior? Why or why not?

3. **(Numerical Methods)** Because solutions to chaotic systems are so sensitive to changes in the initial conditions, it is difficult to get accurate results using numerical methods. In this problem we will consider the Lorenz equations we looked at in class (with R=28) and compare solutions generated using `ode45` and `rk4`.
   (a) Use `ode45` to generate an approximate solution [t,y] on the interval where $0 \leq t \leq 10$ with initial condition [2,3,5]. Use the command `plot(t,y(:,1))` to look at the (approximate) graph of $y_1(t)$.
   (b) Now use `rk4` to generate another approximate solution [s,z], also on the interval where $0 \leq t \leq 10$ with initial condition [2,3,5]. How small must the step size be for the two solutions to (nearly) agree on the interval $[0, 9]$?