21-124 MODELING WITH DIFFERENTIAL EQUATIONS

HOMEWORK 4: DUE IN CLASS ON MARCH 12

1. Consider the system

$$\frac{dP}{dt} = \alpha P\left(1 - \frac{P}{\beta}\right) - \gamma PQ$$
$$\frac{dQ}{dt} = \delta Q\left(1 - \frac{Q}{\epsilon}\right) - \zeta PQ$$

where $\alpha = 2$, $\beta = 7$, $\gamma = 1$, $\delta = 6$, $\epsilon = 9$, $\zeta = 2$. which represents the growth of two populations of animals (Pigeons and Quails, perhaps).

(a) What are the assumptions on which this model is based?
(b) Describe the roll played by each of the parameters. (The six greek letters.)
(c) How would you describe the interaction of these two populations? Are they competing or cooperating? Is it a predator/prey relationship? Is there some other type of interaction?
(d) What are the equilibrium points of the system? (Compute them by hand.)
(e) Use pplane to produce the phase plane for the system. Experiment by plotting a bunch of solutions. How many non-equilibrium solutions can you find? Turn in a printout showing one or two of each type. (Enough to give an idea what is going on, but not so many that it looks crowded and confusing.)
(f) What does this model predict for the future of these two populations? Does it depend on the initial conditions? If so, How?

2. Consider the system

$$\frac{dP}{dt} = \alpha P\left(1 - \frac{P}{\beta}\right) + \gamma PQ$$
$$\frac{dQ}{dt} = \delta Q\left(1 - \frac{Q}{\epsilon}\right) + \zeta PQ$$

where $\alpha = 2$, $\beta = 2$, $\gamma = 0.1$, $\delta = 6$, $\epsilon = 3$, $\zeta = 0.2$. which represents the growth of two populations of animals (Pelicans and Q...?).

(a) What are the assumptions on which this model is based?
(b) Describe the roll played by each of the parameters. (The six greek letters.)
(c) How would you describe the interaction of these two populations? Are they competing or cooperating? Is it a predator/prey relationship? Is there some other type of interaction?
(d) What are the equilibrium points of the system? (Compute them by hand.)
(e) Use pplane to produce the phase plane for the system. Experiment by plotting a bunch of solutions. How many non-equilibrium solutions can you find? Turn in a printout showing one or two of each type. (Enough to give an idea what is going on, but not so many that it looks crowded and confusing.)

(f) What does this model predict for the future of these two populations? Does it depend on the initial conditions? If so, How?

3. Consider the system

$$\frac{dP}{dt} = 5P\left(1 - \frac{P}{2}\right) - PQ - h$$
$$\frac{dF}{dt} = -Q + 2PQ$$

which represents the growth of two populations of animals, one of which preys upon the other. The parameter h represents hunting at a constant rate.

(a) What are the assumptions on which this model is based? Which is the predator, and which is the prey?
(b) What are the equilibrium points of the system? (Compute them by hand. Do they depend on h? If so, how?)
(c) Use pplane to produce the phase plane for the system. Experiment by plotting a bunch of solutions. How many non-equilibrium solutions can you find? Try this for several different values of h between 0 and 0.5. How does changing h affect the equilibrium points?
(d) Choose two values of h, and turn in a printout for each. The printouts should show several different types of solution.
(e) What does this model predict for the future of these two populations? Does it depend on the initial conditions? If so, How?
(f) How can you use this model to explain the increase in the shark population in the Mediteranean Sea during World War II? (Hint: Sharks eat the same fish people do, and naval warfare tends to decrease the size of fishing fleets.)