21-124 MODELING WITH DIFFERENTIAL EQUATIONS

HOMEWORK 2: DUE IN CLASS ON FEBRUARY 12

1. In this problem, we will consider the initial value problem
$$\frac{dy}{dt} = (1+y^2)\cos(t); \quad y(0) = 0.$$
   (a) Find the exact solution (analytically).
   (b) Use the MATLAB routine `eul` to produce an approximate solution on the interval $[0, 6]$, with step size `h`=1. Determine the maximum error for this approximation by comparing witht the exact solution. Repeat this eight times, halving the step size each time. Produce a log-log plot showing maximum error vs. step size.
   (c) Repeat part 1b using `rk2` instead of `eul`.
   (d) Repeat part 1b using `rk4` instead of `eul`.
   (e) Use `Matlab` to display on a single diagram the plots of the exact solution and the approximate solutions generated by each of the three methods using a step size of `h`=.25. Use a distinctive marking for each method. (Note that when you print the graph it will appear in black and white, so simply choosing different colors may not work very well.)
2. A function of the form $y = ax^b$ is called a power function. Use `Matlab`'s `plot` command to sketch the plot of each of the power functions on the interval $[0, 2]$:
   (a) $y = 2x^3$
   (b) $y = 200x^4$
   (c) $y = 50x^{-2}$

   Then use the `loglog` command to produce a log-log plot on the same interval.

   Describe what the graph of a power function looks like when drawn on a `loglog` graph.
3. The accuracy of any numerical method in solving a differential equation of the form $y' = f(t, y)$ depends on how strongly the equation depends on the variable y. (The error bounds depend on the derivatives of f with respect to y.) To see this experimentally, consider the two initial value problems
$$y' = y; \quad y(0) = 1$$
   and
$$y' = e^t; \quad y(0) = 1.$$
   Note that $y(t) = e^t$ is the solution to both problems.
   (a) Use `eul` to compute approximate solutions to the two initial value problems on $[0, 1]$ using a step size of `h`=.01. Compare the accuracy of the solutions to the two problems.
   (b) Repeat this process for the routines `rk2` and `rk4`.