

Homework #5

1. Determine the set of natural numbers n for which the following inequality holds:

$$5^n + 6^n < 7^n.$$

State your claim and prove it using induction.

2. Use induction to prove that for every $n \in \mathbb{N}$, the number of subsets of $[n]$ is exactly 2^n .
3. For $n \in \mathbb{N} \cup \{0\}$, let f_n denote the n th Fibonacci number.

- a. Determine the values of n for which $2n < f_n$. Prove your claim.
- b. Prove that

$$\sum_{k=0}^n f_{2k} = f_{2n+1} - 1.$$

4. In class we proved that if n is a multiple of 3, then f_n is even. Prove the converse of this statement. That is, prove that if n is not a multiple of 3, then f_n is odd.
5. Prove that for every $n \in \mathbb{N}$ we have

$$\sum_{k=1}^n k^3 = \left(\sum_{k=1}^n k \right)^2.$$