Homework #2

1. Let A, B, and C be sets. Prove that

   $$A - (B - C) \subseteq (A - B) \cup C$$

   and then provide an example of sets A, B, and C for which the containment is *strict*.

2. Let A and B be sets, and suppose that $\mathcal{P}(A) = \mathcal{P}(B)$. Is it necessarily the case that $A = B$? If so, prove it. If not, provide a counterexample.

3. For each $n \in \mathbb{N}$, let $A_n = [n] \times [n]$. Define $B = \bigcup_{n\in\mathbb{N}} A_n$. Does $B = \mathbb{N} \times \mathbb{N}$? Either prove that it does, or show why it does not.

4. Let $I = \{x \in \mathbb{R} \,|\, 0 < x < 1\}$. For each $x \in I$, define $S_x = \{y \in \mathbb{R} \,|\, x < y < x + 1\}$. Provide a double containment proof that

   $$\bigcup_{x\in I} S_x = \{z \in \mathbb{R} \,|\, 0 < z < 2\}.$$

5. Prove or disprove each of the following statements:

   $(i.)\ \bigcup_{n\in\mathbb{N}} \mathcal{P}([n]) \subseteq \mathcal{P}(\mathbb{N})$ $\qquad$ $(ii.)\ \mathcal{P}(\mathbb{N}) \subseteq \bigcup_{n\in\mathbb{N}} \mathcal{P}([n])$