

Homework #11

1.
 - a. Let $d = \gcd(1819, 3587)$. Find d using the Euclidean algorithm.
 - b. Use the extended Euclidean algorithm to find $x, y \in \mathbb{Z}$ such that $1819x + 3587y = d$.
2. Find all solutions $x \in \mathbb{Z}$ to the following congruences, or say why none exist.
 - i. $5x \equiv 1 \pmod{12}$.
 - ii. $6x \equiv 1 \pmod{27}$.
 - iii. $56x \equiv 4 \pmod{210}$.

3. Fix $n \in \mathbb{N}$. Prove the following identity by counting in two ways.

$$4^n = \sum_{k=0}^n \binom{n}{k} 3^k$$

4. Consider the word MILLIMETER.
 - a) How many anagrams of MILLIMETER are there?
 - b) How many such anagrams have the two M's adjacent?
 - c) How many such anagrams have the two M's non-adjacent?

5. Fix $n \in \mathbb{N}$. Suppose $A \subseteq \mathbb{Z}$ and A has n elements. Prove there exists a non-empty subset $X \subseteq A$ such that n divides the sum of the elements in X .