Homework #3

1. Consider the following proposition P:

   $$(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x^2 - y^2 \geq 0)$$

   Write $\neg P$ in *positive form*, that is, write down a statement logically equivalent to $\neg P$ with the negation symbol inside the quantifiers (or, better yet, with no negation symbol). Then determine if P or $\neg P$ is true. If P is true, prove it. If $\neg P$ is true, then prove $\neg P$.

2. Write out the following statements symbolically in positive form and determine whether they are true or false (no proof required).

   - There is no real number whose squared is -1.
   - If an integer n has a multiplicative inverse in the integers, then n must be 0 or 1.
   - For any real numbers x and y, if x and y are both nonpositive then their product is nonnegative.
   - The product of two odd integers is not even.

3. Let P denote the set of strictly positive real numbers, that is

   $$P = \{x \in \mathbb{R} | x > 0\}.$$

   For every $x \in P$, define the set S_x as follows

   $$S_x = \{z \in \mathbb{R} | -x \leq z \leq x\}.$$

   Prove the following propositions:

   a.) $(\forall x, y \in P)(S_x \subseteq S_y \Leftrightarrow x \leq y)$

   b.) $\bigcap_{x \in P} S_x = \{0\}$

4. Use a chain of logical equivalences to prove the following propositions.

   a.) Given a universal set U and sets $A, B \subseteq U$, it is the case that $(A \cup B) \cap \overline{A} = B - A$.

   b.) For all sets A, B, and C, it is the case that $A \cap (B - C) = (A \cap B) - (A \cap C)$.