Homework #1

1. Rewrite the following sentences, using set-builder notation to define the set. Then, if possible, write out the set in roster notation. If this can't be done, explain why and write out three elements from the set.

   a.) Let A be the set of all natural numbers whose squares are less than 39.

   b.) Let B be the set of all real numbers that are roots of the equation $x^2 - 3x - 10 = 0$.

   c.) Let C be the set of ordered pairs of integers whose sum is non-negative.

   d.) Let D be the set of ordered pairs of real numbers whose first coordinate is rational and whose second coordinate is irrational.

2. Let $I = \{-1, 0, 1\}$. For each $i \in I$, define $A_i = \{i-2, i-1, i, i+1, i+2\}$ and $B_i = \{-2i, -i, i, 2i\}$. Write out the following sets in roster notation (no justification is required):

   a.) $\bigcup_{i\in I} A_i$ and $\bigcap_{i\in I} A_i$

   b.) $\bigcup_{i\in I} B_i$ and $\bigcap_{i\in I} B_i$

   c.) $(\bigcup_{i\in I} A_i) - (\bigcup_{i\in I} B_i)$ and $(\bigcap_{i\in I} A_i) - (\bigcap_{i\in I} B_i)$

   d.) $\bigcup_{i\in I}(A_i - B_i)$ and $\bigcap_{i\in I}(A_i - B_i)$

3. Let $(a, b) \in \mathbb{R}^2$ and fix $\epsilon \in \mathbb{R}$ with $\epsilon > 0$. Define $C_{(a,b),\epsilon}$ as the set of real numbers "within ϵ" of (a, b):

   $$C_{(a,b),\epsilon} = \{(x, y) \in \mathbb{R}^2 | \sqrt{(x-a)^2 + (y-b)^2} < \epsilon\}.$$

   a.) Give a geometric description of $C_{(a,b),\epsilon}$.

   b.) Identify the following sets. Write your answer in the form of $C_{(a,b),\epsilon}$ or as one of the standard sets discussed in class.

      i. $C_{(0,0),1} \cap C_{(0,0),2}$

      ii. $C_{(0,0),1} \cup C_{(0,0),2}$

      iii. $C_{(0,0),1} \cap C_{(2,2),1}$

   c.) For a given $\epsilon > 0$, define $D_{(a,b),\epsilon}$ as follows:

   $$D_{(a,b),\epsilon} = \{(x, y) \in \mathbb{R}^2 | \sqrt{(x-a)^2 + (y-b)^2} \leq \epsilon\}.$$

   What is $D_{(a,b),\epsilon} - C_{(a,b),\epsilon}$ geometrically? Write a definition for this set using set-builder notation.

4. For each $x \in \mathbb{R}$, define the set P_x as follows:

   $$P_x = \{y \in \mathbb{R} | \, y = x^n \text{ for some } n \in \mathbb{N}\}$$

   a.) There are exactly 3 values of x for which P_x is finite. What are they and why?

b.) Determine the sets

$$\bigcap_{0<x<1} P_x \text{ and } \bigcup_{0<x<1} P_x.$$

Provide a brief justification for your answers. (A full proof is not necessary.)

c.) Determine the sets

$$\bigcap_{k\in[3]} P_{2^k} \text{ and } \bigcap_{k\in\mathbb{N}} P_{2^k}.$$

Provide a brief justification for your answers.