Ex: How many ways are there to choose a chief and two hench persons from a group of 10 ppl?

So kWh: - 10 choices for chief
- once chief chosen, (9) choices for hench ppl.

\[
\text{\# of such committees} = 10 \cdot \binom{9}{2}
\]

\[
= 10 \cdot \frac{9!}{7! \cdot 2!}
\]

\[
= 10 \cdot \frac{9 \cdot 8}{2 \cdot 1} = 360
\]

Alternate kWh: first pick 3 people for committee, then check chief from among these:

\[
\text{\# of such committees} = \binom{10}{3}(1)
\]

\[
= 120 \cdot 3 = 360
\]

as before!
Countin' Poker hands

- A standard deck consists of 52 cards.
- Each card has 1 of 4 possible suits:
  \((\heartsuit, \diamondsuit, \spadesuit, \clubsuit)\)

  and 1 of 13 possible ranks:
  \((A, 2, 3, \ldots, 9, 10, J, Q, K, A)\)

  e.g. A\(\heartsuit\) and 9\(\clubsuit\) are cards.

  A poker hand is a 5-selecten from a standard deck.

Ex's 1) How many distinct hands are possible?

Sol'n: \(\binom{52}{5} = 2,598,960\).

2) A full house is a hand consisting of 3 cards of one rank and 2 cards of another, e.g. A\(\diamondsuit\), A\(\heartsuit\), 3\(\spadesuit\), 3\(\diamondsuit\), 3\(\clubsuit\)

  How many distinct full house hands are possible?
\[ \text{Sol'n: - pick two ranks, } \binom{13}{2}, \binom{2}{1}, \binom{3}{2}, \binom{4}{2} = 3,744 \]

3. A 3-of-a-kind consists of 3 cards from a single rank and 2 cards from two other distinct ranks, e.g., 3 Q's, a 10, and a J.

Q: How many 3-of-a-kind hands are possible?

Sol'n: - pick 3-card rank \( \binom{13}{1} \)
- from this rank, pick 3 cards \( \binom{4}{3} \)
- pick remaining two ranks \( \binom{12}{2} \)
- from the first, pick a card \( \binom{4}{1} \)
- also from the second \( \binom{4}{1} \)

\[ \Rightarrow \text{# of 3 of a kind hands} = \binom{13}{1}\binom{4}{1}\binom{12}{2}\binom{4}{1}\binom{4}{1} = 59,912 \]

Alt sol'n: - Pick three ranks \( \binom{13}{3} \)
- from there, pick 3-card rank \( \binom{3}{1} \)
- pick three cards from this rank \( \binom{4}{3} \)
- from other two ranks, pick cards \( \binom{4}{1}\binom{4}{1} \)

But: \( \binom{13}{3}\binom{3}{1}\binom{4}{3}\binom{4}{1}\binom{4}{1} = 59,912 \)

Binary sequence: a binary sequence (of length \( n \)) is an ordered sequence of 0's and 1's (of length \( n \))
e.g. 011 and 101 are binary sequences of length 3.

Let $P_n$ denote the set of binary sequences of length $n$.

Ex: 1. What is $|P_n|$?
    
2. How many sequences $s \in P_n$ have at least two 1's? (assuming $n \geq 2$)

Sol'n: 1. Each $s \in P_n$ formed by making a sequence of $n$ choices:

\[
\begin{array}{cccc}
0 & 1 & 0 & 1 \\
\hline \\
\end{array}
\]

length $n$ \Rightarrow $|P_n| = 2 \cdot 2 \cdots 2$

2 choices for each space \Rightarrow $|P_n| = 2^n$

2. Easier to count \# of seq's w/ either zero 1's or one 1, then subtract:

\# w/ zero 1's = 1 (just 00...0)
\# w/ one 1 = $n$ (one for each place to put the 1
\[ \Rightarrow \text{# w/ at least two 1's} \]
\[ = 2^n - n - 1 \]

**Theorem:** Fix \( n \in \mathbb{N} \cup \{0\} \)

Then: 
\[ 2^n = \binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n} \]
\[ = \sum_{k=0}^{n} \binom{n}{k} \]

**Pf.** If \( n=0 \), then 
\[ 2^0 = 1 = \binom{0}{0} = \sum_{k=0}^{0} \binom{n}{k} \]
So suppose \( n \geq 1 \).

We proved \( |P_n| = 2^n \)

We can partition \( P_n = S_0, uS_1, uS_2, \ldots, uS_n \)

where \( S_k = \text{set of sequences w/ exactly } k \text{-many 1's} \).

**Observe:** 
\[ |S_k| = \binom{n}{k} = \text{# of ways to pick } k \text{ positions out of } n \text{ where 1's appear} \]

Hence (by RoS): 
\[ 2^n = |P_n| = |S_0| + |S_1| + \ldots + |S_n| \]
\[ = \binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n} \]
\[ = \sum_{k=0}^{n} \binom{n}{k} \]