Homework #8

1. Define a relation $\preceq$ on $\mathbb{N} \times \mathbb{N}$ by the rule $(n_0, m_0) \preceq (n_1, m_1)$ iff $n_0 \leq n_1$ and $m_0 \leq m_1$. Prove that $\preceq$ is a partial order on $\mathbb{N} \times \mathbb{N}$. Provide an example to show that $\preceq$ is not a total order.

2. Let A be a set and suppose R is an equivalence relation on A. Prove that set of equivalences classes, A/R, is a partition of A.

3. Let A be a set and suppose R is a partial order on A (that is, R is a reflexive, transitive, and anti-symmetric relation on A). For $x \in A$ define the *cone of* x, denoted $\langle x \rangle_R$, as follows

   $$\langle x \rangle_R = \{a \in A \,|\, (a, x) \in R\}$$

   Prove that for all $x, y \in A$, we have $\langle x \rangle_R \subseteq \langle y \rangle_R$ if and only if $(x, y) \in R$.