Homework #5

1. Use induction to prove that for every $n \in \mathbb{N}$ we have that $(2n+1)^2 - 1$ is a multiple of 8.
2. Determine the set of natural numbers n for which the following inequality holds:

$$5^n + 6^n < 7^n.$$

   State your claim and prove it using induction.

3. Use induction to prove that for every $n \in \mathbb{N}$ we have

$$\sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k \right)^2.$$