Homework #3

1. Prove or disprove each of the following statements:

$$(i.)\ \bigcup_{n\in\mathbb{N}} \mathcal{P}([n]) \subseteq \mathcal{P}(\mathbb{N}) \qquad\qquad (ii.)\ \mathcal{P}(\mathbb{N}) \subseteq \bigcup_{n\in\mathbb{N}} \mathcal{P}([n])$$

2. *For this problem, you may use the fact that* $\frac{1}{n}$ *gets arbitrarily close to* 0 *as* n *gets larger and larger. That is, you may assume that for every* $z \in \mathbb{R}$ *with* $z > 0$*, there is an* $n \in \mathbb{N}$ *such that* $\frac{1}{n} < z$*.*

   For each $n \in \mathbb{N}$, define the sets A_n and B_n as follows:

$$A_n = \left\{ x \in \mathbb{R} \,|\, 0 \leq x \leq \frac{n-1}{n} \right\}$$

$$B_n = \left\{ y \in \mathbb{R} \,|\, -\frac{1}{n} < y < 1 \right\}.$$

   Use a double containment argument to prove that

$$\bigcup_{n\in\mathbb{N}} A_n = \bigcap_{n\in\mathbb{N}} B_n.$$

3. Consider the following proposition P:

$$(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x^2 - y^2 \geq 0)$$

   Write $\neg P$ in *positive form*, that is, write down a statement logically equivalent to $\neg P$ with the negation symbol inside the quantifiers (or, better yet, with no negation symbol). Then determine if P or $\neg P$ is true. If P is true, prove it. If $\neg P$ is true, then prove $\neg P$.

4. Write out the following statements symbolically in positive form and determine whether they are true or false (no proof required).

   - There is no real number whose square is -1.
   - If an integer n has a multiplicative inverse in the integers, then n must be 0 or 1.
   - For any real numbers x and y, if x and y are both nonpositive then their product is nonnegative.
   - The product of two odd integers is not even.