

Homework #14

1. A *standard deck* of cards consists of 52 cards. Each card is designated by one of the 4 possible suits $\heartsuit, \clubsuit, \diamondsuit, \spadesuit$, and one of the 13 possible ranks 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A, listed here in ascending order. A poker hand is a 5-selection from a standard deck.
 - a. A *flush* is a hand consisting of 5 cards of the same suit, which are not of consecutive rank. For example, $5\diamondsuit, J\diamondsuit, Q\diamondsuit, 2\diamondsuit, 9\diamondsuit$ is a flush. How many distinct flushes are there?
 - b. A *straight* is a hand consisting of 5 cards of consecutive rank, which are not all of a single suit. For example, $8\diamondsuit, 9\clubsuit, 10\clubsuit, J\spadesuit, Q\diamondsuit$ is a straight. A straight can have an A as its high card or low card, but not a middle card. So $10\diamondsuit, J\heartsuit, Q\clubsuit, K\clubsuit, A\diamondsuit$ and $A\diamondsuit, 2\spadesuit, 3\diamondsuit, 4\spadesuit, 5\clubsuit$ are straights, but $Q\clubsuit, K\clubsuit, A\diamondsuit, 2\spadesuit, 3\diamondsuit$ is not. How many distinct straights are there?
 - c. A *straight flush* is a hand consisting of 5 cards of consecutive rank and of the same suit. For example, $8\diamondsuit, 9\diamondsuit, 10\diamondsuit, J\diamondsuit, Q\diamondsuit$ is a straight flush. How many distinct straight flushes are there?
2. A student organization holds meetings every week, with one chosen leader and two assistants to run the meeting efficiently. If there are 14 weeks in a semester, how many students must be in the organization to guarantee that they can have a different set of leaders/assistants at every meeting?
3. Fix $n \in \mathbb{N}$. Prove the following identity by counting in two ways.

$$4^n = \sum_{k=0}^n \binom{n}{k} 3^k$$

4. Consider the word MILLIMETER.
 - a) How many anagrams of MILLIMETER are there?
 - b) How many such anagrams have the two M's adjacent?
 - c) How many such anagrams have the two M's non-adjacent?
5. Fix $n \in \mathbb{N}$. Suppose $A \subseteq \mathbb{Z}$ and A has n elements. Prove there exists a non-empty subset $X \subseteq A$ such that n divides the sum of the elements in X .