Homework #10

1. Define functions $f : \mathbb{N} \to \mathbb{N}$ and $g : \mathbb{N} \to \mathbb{N}$ such that $g \circ f = \mathrm{id}_{\mathbb{N}}$ but $f \circ g \neq \mathrm{id}_{\mathbb{N}}$. Prove that your functions satisfy these identities.

2. Suppose that A and B are sets. Suppose further that $|A| = |B|$, that is, there exists a bijection $f : A \to B$. Show that $|\mathcal{P}(A)| = |\mathcal{P}(B)|$ by constructing a bijection $F : \mathcal{P}(A) \to \mathcal{P}(B)$. Prove that the function F you construct is a bijection.

3. a. Define a function $f : \mathbb{R}^3 \to \mathbb{R}^2$ by $f(x, y, z) = (xy, xz)$. Prove or disprove the following statements.
      - f is injective.
      - f is surjective.

   b. Define $f : \mathbb{R}^2 \to \mathbb{R}^2$ by the rule $f(x, y) = (3x + 2y, 4x + y)$. Show that f is a bijection by explicitly defining an inverse function for f and proving it is the inverse.