Homework #1

1. Rewrite the following sentences, using set-builder notation to define the set. Then, if possible, write out the set in roster notation.

   a.) Let A be the set of all natural numbers whose squares are strictly less than 39.

   b.) Let B be the set of all real numbers that are roots of the equation $x^2 - 3x - 10 = 0$.

2. Let $I = \{-1, 0, 1\}$. For each $i \in I$, define $A_i = \{i-2, i-1, i, i+1, i+2\}$ and $B_i = \{-2i, -i, i, 2i\}$. Write out the following sets in roster notation (no justification is required):

   a.) $\bigcup_{i\in I} A_i$ and $\bigcap_{i\in I} A_i$

   b.) $\bigcup_{i\in I} B_i$ and $\bigcap_{i\in I} B_i$

   c.) $(\bigcup_{i\in I} A_i) - (\bigcup_{i\in I} B_i)$ and $(\bigcap_{i\in I} A_i) - (\bigcap_{i\in I} B_i)$

   d.) $\bigcup_{i\in I}(A_i - B_i)$ and $\bigcap_{i\in I}(A_i - B_i)$

3. For each $x \in \mathbb{R}$, define the set P_x as follows:

   $$P_x = \{y \in \mathbb{R} \,|\, y = x^n \text{ for some } n \in \mathbb{N}\}$$

   a.) There are exactly 3 values of x for which P_x is finite. What are they and why?

   b.) Determine the sets

   $$\bigcap_{0<x<1} P_x \text{ and } \bigcup_{0<x<1} P_x.$$

   Provide a brief justification for your answers. (A full proof is not necessary.)

   c.) Determine the sets

   $$\bigcap_{k\in[3]} P_{2^k} \text{ and } \bigcap_{k\in\mathbb{N}} P_{2^k}.$$

   Provide a brief justification for your answers. (A full proof is not necessary.)