

Homework #8

1. Define a relation \preceq on $\mathbb{N} \times \mathbb{N}$ by the rule $(n_0, m_0) \preceq (n_1, m_1)$ iff $n_0 \leq n_1$ and $m_0 \leq m_1$. Prove that \preceq is a partial order on $\mathbb{N} \times \mathbb{N}$. Provide an example to show that \preceq is not a total order.
2. Let A be a set and suppose R is an equivalence relation on A . Prove that set of equivalence classes, A/R , is a partition of A .
3. Let A be a set and suppose R is a partial order on A (that is, R is a reflexive, transitive, and anti-symmetric relation on A). For $x \in A$ define the *cone of x* , denoted $\langle x \rangle_R$, as follows

$$\langle x \rangle_R = \{a \in A \mid (a, x) \in R\}$$

Prove that for all $x, y \in A$, we have $\langle x \rangle_R \subseteq \langle y \rangle_R$ if and only if $(x, y) \in R$.