## Homework \#5

1. Use induction to prove that for every $n \in \mathbb{N}$ we have that $(2 n+1)^{2}-1$ is a multiple of 8 .
2. Determine the set of natural numbers $n$ for which the following inequality holds:

$$
5^{n}+6^{n}<7^{n}
$$

State your claim and prove it using induction.
3. Use induction to prove that for every $n \in \mathbb{N}$ we have

$$
\sum_{k=1}^{n} k^{3}=\left(\sum_{k=1}^{n} k\right)^{2}
$$

