Homework #14

1. Recall from class: a standard deck consists of 52 cards. Each card is designated by one of the 4 possible suits $\heartsuit, \clubsuit, \diamondsuit, \spadesuit$, and one of the 13 possible ranks 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A, listed here in ascending order. A poker hand is a 5-selection from a standard deck.

   a. A *flush* is a hand consisting of 5 cards of the same suit, which are not of consecutive rank. For example, $5\diamondsuit$, $J\diamondsuit$, $Q\diamondsuit$, $2\diamondsuit$, $9\diamondsuit$ is a flush. How many distinct flushes are there?

   b. A *straight* is a hand consisting of 5 cards of consecutive rank, which are not all of a single suit. For example, $8\diamondsuit$, $9\clubsuit$, $10\clubsuit$, $J\spadesuit$, $Q\diamondsuit$ is a straight. A straight can have an A as its high card or low card, but not a middle card. So $10\diamondsuit$, $J\heartsuit$, $Q\clubsuit$, $K\clubsuit$, $A\diamondsuit$ and $A\diamondsuit$, $2\spadesuit$, $3\diamondsuit$, $4\spadesuit$, $5\clubsuit$ are straights, but $Q\clubsuit$, $K\clubsuit$, $A\diamondsuit$, $2\spadesuit$, $3\diamondsuit$ is not. How many distinct straights are there?

   c. A *straight flush* is a hand consisting of 5 cards of consecutive rank and of the same suit. For example, $8\diamondsuit$, $9\diamondsuit$, $10\diamondsuit$, $J\diamondsuit$, $Q\diamondsuit$ is a straight flush. How many distinct straight flushes are there?

2. A student organization holds meetings every week, with one chosen leader and two assistants to run the meeting efficiently. If there are 14 weeks in a semester, how many students must be in the organization to guarantee that they can have a different set of leaders/assistants at every meeting?

3. Fix $n \in \mathbb{N}$. Prove the following identity by counting in two ways.

$$4^n = \sum_{k=0}^{n} \binom{n}{k} 3^k$$

4. Consider the word MILLIMETER.

   a) How many anagrams of MILLIMETER are there?

   b) How many such anagrams have the two M's adjacent?

   c) How many such anagrams have the two M's non-adjacent?

5. Fix $n \in \mathbb{N}$. Suppose $A \subseteq \mathbb{Z}$ and A has n elements. Prove there exists a non-empty subset $X \subseteq A$ such that n divides the sum of the elements in X.