Homework #12

1. Fix $a, b \in \mathbb{Z}$, not both 0, and $m \in \mathbb{Z}$. Prove that $gcd(a,b) = \gcd(a+bm, b)$ by proving $gcd(a,b) \leq \gcd(a+bm,b)$ and $gcd(a,b) \geq \gcd(a+bm,b)$.

2. Fix $a, b \in \mathbb{Z}$, not both 0, and $m \in \mathbb{N}$. Prove that $\gcd(am, bm) = m\gcd(a,b)$.

   (One approach: prove $\gcd(am, bm) \leq m\gcd(a,b)$ using Bezout's theorem, and $\gcd(am, bm) \geq m\gcd(a,b)$ directly.)

3. Fix $a, b \in \mathbb{Z}$ and suppose that $\gcd(a,b) = 1$. Prove that $\gcd(a+b, a-b)$ is either 1 or 2.

4. Fix $p \in \mathbb{N}$ a prime, with $p > 3$. Prove that $p^2 \equiv 1 \pmod{24}$.