

# Homework #10

1. Define functions  $f : \mathbb{N} \rightarrow \mathbb{N}$  and  $g : \mathbb{N} \rightarrow \mathbb{N}$  such that  $g \circ f = \text{id}_{\mathbb{N}}$  but  $f \circ g \neq \text{id}_{\mathbb{N}}$ . Prove that your functions satisfy these identities.
2. Suppose that  $A$  and  $B$  are sets. Suppose further that  $|A| = |B|$ , that is, there exists a bijection  $f : A \rightarrow B$ . Show that  $|\mathcal{P}(A)| = |\mathcal{P}(B)|$  by constructing a bijection  $F : \mathcal{P}(A) \rightarrow \mathcal{P}(B)$ . Prove that the function  $F$  you construct is a bijection.
3.
  - a. Define a function  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  by  $f(x, y, z) = (xy, xz)$ . Prove or disprove the following statements.
    - $f$  is injective.
    - $f$  is surjective.
  - b. Define  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  by the rule  $f(x, y) = (3x + 2y, 4x + y)$ . Show that  $f$  is a bijection by explicitly defining an inverse function for  $f$  and proving it is the inverse.