Homework #1 (edited)

1. Rewrite the following sentences, using set-builder notation to define the set. Then, if possible, write out the set in roster notation. If this can't be done, explain why and write out three elements from the set.

   a.) Let A be the set of all natural numbers whose squares are less than 39.

   b.) Let B be the set of all real numbers that are roots of the equation $x^2 - 3x - 10 = 0$.

2. Let $I = \{-1, 0, 1\}$. For each $i \in I$, define $A_i = \{i-2, i-1, i, i+1, i+2\}$ and $B_i = \{-2i, -i, i, 2i\}$. Write out the following sets in roster notation (no justification is required):

   a.) $\bigcup_{i\in I} A_i$ and $\bigcap_{i\in I} A_i$

   b.) $\bigcup_{i\in I} B_i$ and $\bigcap_{i\in I} B_i$

   c.) $(\bigcup_{i\in I} A_i) - (\bigcup_{i\in I} B_i)$ and $(\bigcap_{i\in I} A_i) - (\bigcap_{i\in I} B_i)$

   d.) $\bigcup_{i\in I}(A_i - B_i)$ and $\bigcap_{i\in I}(A_i - B_i)$

3. For each $x \in \mathbb{R}$, define the set P_x as follows:

   $$P_x = \{y \in \mathbb{R} |\, y = x^n \text{ for some } n \in \mathbb{N}\}$$

   a.) There are exactly 3 values of x for which P_x is finite. What are they and why?

   b.) Determine the sets

   $$\bigcap_{0<x<1} P_x \text{ and } \bigcup_{0<x<1} P_x.$$

   Provide a brief justification for your answers. (A full proof is not necessary.)

   c.) Determine the sets

   $$\bigcap_{k\in[3]} P_{2^k} \text{ and } \bigcap_{k\in\mathbb{N}} P_{2^k}.$$

   Provide a brief justification for your answers.