

Measure Theory and Lebesgue Integration.

Gautam Iyer, Fall 2014

- L1, Mon 8/25.* • Introduction and motivation.
 • Lebesgue Measure on \mathbb{R}^n .
 – [1, §13] Abstract σ -algebras and measures.
- L2, Wed 8/27.* – [1, §11] Lebesgue outer measure.
- L3, Fri 8/29.* – [1, §12] Separated additivity, and the outer measure of cells.
- L4, Wed 9/03.* – [1, §13; 2, §1.3] The Caratheodory condition.
 – [1, §13] The Lebesgue σ -algebra and uniqueness.
- L5, Fri 9/5.* • Abstract measures
 – [2, §1.6] Dynkin systems, and equality of finite measures.
- L6, Mon 9/8.* – [2, §1.5; 3, §7.2] Regularity of measures.
- L7, Wed 9/10.* – [2, §1.4] Non Lebesgue-measurable sets
- L8, Fri 9/12.* – [2, §1.5; 3, §1.3] Completion
- L9, Mon 9/15.* • Measurable functions
 – [2, §2.1; 3, §2.1] Definition and elementary properties.
- L10, Wed 9/17.* – [2, §2.1] Cantor function & non-Borel sets.
- L11, Fri 9/19.* – [2, §2.1; 3, §2.4] Approximation results.
- L12, Mon 9/22.* • Integration.
 – [2, §2.3; 3, §2.2] Construction of the integral.
 – [2, §2.4; 3, §2.2] Monotone convergence and linearity.
- L13, Wed 9/24.* – [2, §2.4; 3, §2.2] Fatou's lemma and dominated convergence.
- L14, Fri 9/26.* – [2, §2.6] Push forward measures.
 • Convergence
 – [2, §3.1] Egorov's theorem
- L15, Mon 9/29.* – [2, §3.1] Convergence almost everywhere and in measure.
 – L^p spaces.
 * [2, §3.2] Normed vector spaces and Banach spaces.
- L16, Wed 10/1.* * [5, §3] Hölder's inequality and duality.
- L17, Mon 10/6.* * [2, §3.3; 5, §3] Completeness and Convergence.
- L18, Wed 10/8.* – Uniform integrability, Vitali convergence theorem.
- L19, Mon 10/13.* • Signed Measures
 – [2, §4.1; 3, §3.1] Hanh and Jordan decompositions.
- L20, Wed 10/15.* – [2, §4.1; 3, §3.1] The total variation norm of measures.

- L21, Mon 10/20.* – [2, §4.2; 3, §3.2] Radon Nikodym theorem.
- L22, Wed 10/22.* – [2, §4.3; 3, §3.2] Lebesgue Decomposition
- L23, Fri 10/24.* – [2, §3.5, §4.5; 3, §6.2] The dual of L^p .
- L24, Mon 10/27.* • [2, §7; 3, §7.1, 7.3] Riesz Representation Theorem.
 – Construction of μ .
- L25, Wed 10/29.* – Proof of the representation.
- L26, Fri 10/31.* • Integration on Product Spaces.
 – [2, §5.1, 5.2; 3, §2.5] The product measure.
- L27, Mon 11/3.* – [2, §5.2, 5.3; 3, §2.5] Fubini and Tonelli theorems.
- L28, Wed 11/5.* • Convolutions
 – [5, §7] Young's inequality
 – [5, §7] L^p -convergence of approximate identities.
- L29, Fri 11/7.* • Fourier Series.
 – [6, §1] Dirichlet, Fejér kernels and convergence of Cesàro sums.
- L30, Mon 11/10.* – [6, §1] Riemann Lebesgue Lemma Parseval's identity
- L31, Wed 11/12.* – Periodic Sobolev spaces and embedding theorems.
- L32, Fri 11/14.* • Lebesgue Differentiation
 – [2, §6.2; 5, §7] Vitali covering and the Maximal function.
- L33, Mon 11/17.* – [2, §6.2; 5, §7] Lebesgue points and differentiation of measures.
- L34, Wed 11/19.* – [2, §6.3; 5, §7] Absolute continuity of functions of one variable.
- L35, Fri 11/21.* – [2, §6.3; 5, §7] Fundamental theorem of Calculus.
- L36, Mon 11/25.* – [2, §6.1; 5, §7] Change of variable.
- L37, Mon 12/2.* • Fourier Transform
 – [3, §8.3; 5, §9] L^1 theory and inversion.
- L38, Wed 12/4.* – [3, §8.3; 5, §9] Plancherel theorem and L^2 -theory.
- L39, Fri 12/6.* – [3, §9.3] Sobolev spaces and embeddings.

References

- [1] R. G. Bartle, *The elements of integration and Lebesgue measure*, Wiley Classics Library, John Wiley & Sons, Inc., New York, 1995. Containing a corrected reprint of the 1966 original [It The elements of integration, Wiley, New York; MR0200398 (34 #293)], A Wiley-Interscience Publication. MR1312157 (95k:28001)
- [2] D. L. Cohn, *Measure theory*, Birkhäuser, Boston, Mass. 1980. MR578344 (81k:28001)
- [3] G. B. Folland, *Real analysis*, Second, Pure and Applied Mathematics (New York), John Wiley & Sons, Inc., New York, 1999. Modern techniques and their applications, A Wiley-Interscience Publication. MR1681462 (2000c:00001)
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- [6] W. Schlag, *Lecture notes on harmonic analysis*, 2005. Available online at http://www.math.uchicago.edu/~schlag/harmonicnotes_old.pdf.